

Group Work 6 - Math 165

Names: _____

Turn in one copy of your work and answers with all names of group members on it. Show all work to receive credit for this exercise. You may not consult with other groups, but you may use your book and notes, and ask questions of the instructor. This exercise counts as one quiz score.

1. Decide whether the following statements are true or false. If they are true, justify your assertion with an argument. If they are false, give a counterexample.

(a) If f is continuous and $f(x) \geq 0$ for all x in $[a, b]$, then $\int_a^b f(x) dx \geq 0$.

(b) If $\int_a^b f(x) dx = 0$, then $f(x) = 0$ for all x in $[a, b]$.

(c) If $G(x) = \int_a^x f(t) dt$ is continuous, then $f(x)$ is continuous.

(d) If F is an antiderivative of f on $[a, b]$, then F is continuous on $[a, b]$.

2. Consider the function $f(x) = |x|$ on $[-2, 2]$. Find the average value of f on $[-2, 2]$, and then find the point c in $[-2, 2]$ that satisfies the result of the Mean Value Theorem for Integrals.

3. Prove (by a substitution) that $\int_a^b f(-x) dx = \int_{-b}^{-a} f(x) dx$.

4. Define a function $L(x) = \int_1^x \frac{1}{t} dt$.

(a) What is the value of $L(1)$?

(b) What is $D_x[L(x)]$? (Assume $x > 0$)

(c) Find $D_x[L(x^3 - x + 4)]$.

5. Does there exist a function f such that $\int_0^x f(t) dt = x + 1$? Justify your answer.

6. If $g(x)$ is an even function, what can be said about $g'(x)$?