

Math 515
Real Analysis
Problem Set 7

You may consult with other human beings on these problems

Due date: November 10, 2005

Each problem is worth 10 points unless otherwise stated.

1. Show that the completion of a separable metric space is separable. Show that the Cartesian product of two separable metric spaces is separable and conversely.
2. Show that the space of bounded sequences of real numbers (functions on $X = \{1, 2, 3, \dots\}$) is not separable.
3. Show that the continuous image of a compact metric space into another metric space is compact. Show that a continuous function from a compact metric space into the real line is bounded and attains its bounds.
4. Prove that if A, B are closed disjoint subsets of a metric space and one of them, (say A), is compact, then $\rho(A, B) > 0$.
5. If A, B are nowhere dense in X , then so is $A \cup B$. If A is nowhere dense in X and B is nowhere dense in Y , then $A \times B$ is nowhere dense in $X \times Y$.
6. If X, Y are first category, so is $X \times Y$.
7. Definition: A point $x \in X$ is isolated if it is not a limit point of X . Prove that a space of the first category has no isolated points. Prove that a complete, countable metric space must have isolated points.