

Math 515
Real Analysis
Problem Set 6

You may consult with other human beings on these problems

Due date: November 3, 2005

Each problem is worth 10 points unless otherwise stated.

1. Give an example of a continuous function which is not uniformly continuous. (Support your assertions with proof!)
2. Show that every metric space is homeomorphic to a metric space of diameter no more than unity. (Hint use a problem from Problem set 5.)
3. Suppose X, Y, Z are metric spaces. Suppose $f : X \rightarrow Y$, $g : Y \rightarrow Z$ are continuous. Prove that $g \circ f$ is continuous. If f, g are uniformly continuous, so is $g \circ f$.
4. Show that a subset of a complete metric space is complete in the induced metric if and only if it is a closed set.
5. Show that the space of bounded sequences is complete in the uniform metric.
6. Show that the space of square summable sequences ℓ^2 , is complete.
7. Prove that the product of two metric spaces is complete if and only if both metric spaces are complete.
8. Let X be complete and $\{A_i\}_{i=1}^{\infty}$ be a sequence of nonempty closed subsets of X with the properties that $A_{i+1} \subset A_i$ and $\lim_{i \rightarrow \infty} \text{diam}(A_i) = 0$. Prove that $\bigcap_{i=1}^{\infty} A_i$ consists of exactly one point.