Math 515

Real Analysis

Problem Set 2

Due date: September 16, 2005

Consultation with other class members is encouraged for this problem set.

1. 10 points. Construct a closed, nowhere dense subset of $[0,1]$ which has prescribed positive Lebesgue measure α (the measure we constructed in class) for each $\alpha \in (0,1)$.

2. 5 points. Show that the set of real numbers in $(0,1)$ whose decimal expansion does *not* contain the number 5 has measure zero. (Hint: What is true about the tertary expansion of the elements of the Cantor set.)

3. 5 points. Let $\{E_n\}_{n=1}^{\infty}$ be a decreasing sequence of measurable subsets of a bounded interval, that is $E_{n+1} \subset E_n$ for all n. show that

$$m(\bigcap_{n=1}^{\infty} E_n) = \lim_{n\to\infty} m(E_n)$$

4. 5 points. Prove that $f(x) \equiv \lim_{m\to+\infty} \lim_{n\to+\infty} (\cos(m!\pi x))^{2n}$ is measurable. (Hint: Identify it with a well known function.)

5. 5 points. Show that if $f(\cdot)$ is measurable, so is $|f(\cdot)|$. Is the converse true, why or why not? (Hint: Assume the existence of a nonmeasurable set. One has to invoke the Axiom of Choice to prove that such a set exists.) Use the first result to prove that if f, g are measurable, so are $\max(f,g)$ and $\min(f,g)$.