Math 515

Real Analysis

Problem Set 1

Due date: September 2, 2005

Consultation with other individuals is not permitted for this problem set.

1. (a) (5) pts. Define precisely (by consulting other texts if you need to do so) what is meant by the statement "$f(x)$ is a piecewise continuous function." You may assume the domain of this function is the closed, bounded interval $[a, b]$ and that it is real valued.

(b) 10 pts. Prove that a piecewise continuous function on such an interval is Riemann integrable. (You may assume the result when $f(x)$ is continuous.)

2. 15 pts. Let, for $x \in [0, 1]$,

$$f(x) = \begin{cases} \frac{1}{q} & \text{if } x = \frac{p}{q} \\ 0 & x \text{ is irrational.} \end{cases}$$

Show that $f(x)$ is Riemann integrable and give the value of the integral.

3. 10 pts. Show that if E_1, E_2 are subsets of the real line and E_1 has measure zero while $E_1 \cup E_2$ is measurable, then E_2 is also measurable.

4. 10 pts. Show that $E \subset (a.b)$ is measurable if and only if for every $\varepsilon > 0$ there exist sets F, G with $F \subset E \subset G$, F closed, G open and $m(G) - m(F) < \varepsilon$.