MATH 557, Fall 2007, Homework set # 5 , Due: September 28, 2007

1. Suppose that $q(t)$ is continuous on $[0, \pi$. Let $\varphi(t, \lambda)$ be the unique solution of $y'' + \lambda y = -q(t)y$ with $y(0, \lambda) = y'(0, \lambda - 1 = 0$. Show that on this interval

$$\varphi(t, \lambda) = \frac{\sin(\sqrt{\lambda} t)}{\sqrt{\lambda}} + M_1 \text{ and } \varphi'(t, \lambda) = \cos(\sqrt{\lambda} t) + M_2$$

where $|M_1| \leq K/\lambda$ and $M_2 \leq K/\sqrt{\lambda}$ for some constant K. (Hint: Show that φ satisfies the integral equation

$$\varphi(t, \lambda) = \frac{\sin(\sqrt{\lambda} t)}{\sqrt{\lambda}} + \int_0^t \sin(\sqrt{\lambda}(t-s)) q(s) \varphi(s, \lambda)\, ds,$$

exits on $[0, \pi]$ and estimate the second term. Differentiate both sides of the above integral equation and estimate the derivative of the second term.

2. If $A(t)$ is a differentiable square matrix with inverse $A^{-1}(t)$ on some interval I, show that $A^{-1}(t)$ is differentiable and find a formula for its derivative.

3. Suppose that $A(t)$ is a continuous n^2 square matrix and $g(t)$ is a continuous n vector on the real line. Suppose that these quantities are in L^1 of the line, i. e.

$$A_0 \equiv \int_{-\infty}^{\infty} |A(s)|\, ds < \infty \text{ and } g_0 \equiv \int_{-\infty}^{\infty} |g(s)|\, ds < \infty.$$

Show that for any n vector η, the solution $\varphi(t)$ of the initial value problem for the equation $y'(t) = A(t)y(t) + g(t)$ with $\varphi(t_0) = \eta$ exists on the line and find a bound for $|\varphi(t)|$ in terms of A_0, g_0.