

MATH 557, Fall 2007, Homework set # 3, Due: September 14, 2007

1. Find an integral equation equivalent to the (scalar) initial value problem: $y''' = f(t, y)$ with $y(t_0) = y_0$, $y'(t_0) = y_1$, $y''(t_0) = y_2$. Generalize your answer to the case $y^{(n)} = f(t, y)$ where n is a positive integer.

2. Let $f(x, y) = xe^{-y^2}$ and find L such that

$$|f(x, y) - f(x, \hat{y})| \leq L|y - \hat{y}|$$

in the strip $E = \{(x, y) : 1 < x < 3, -\infty < y < \infty\}$.

3. Compute explicitly the sequence $\{y_n(x)\}_{n=0}^{\infty}$ of successive approximations for the initial value problem

$$y' = ky \quad y(\xi) = \eta$$

where k is a constant. Verify that $\lim_{n \rightarrow \infty} y_n(x)$ exists and gives the correct solution of the initial value problem.