

MATH 557, Fall 2007, Homework set # 2, Due: September 5 , 2007

1. Prove Schwarz's inequality. That is, show that if $\vec{a}, \vec{b} \in \mathbb{E}_n$, then

$$|\vec{a} \cdot \vec{b}| \leq \|\vec{a}\| \|\vec{b}\|$$

where $\vec{a} \cdot \vec{a} = \|\vec{a}\|^2$. Give necessary and sufficient conditions for equality to hold. Hint, if λ is a real number,

$$f(\lambda) \equiv \|\vec{a} + \lambda \vec{b}\|^2 \geq 0$$

is what kind of a function? Expand it out to see.

2. Suppose $A = (a_{ij})$ is an $n \times n$ matrix such that $a_{ij} = 0$ if $i > j$. Discuss how to solve the system $\vec{y}' = A\vec{y}$ for $\eta = (\eta_0, \eta_1, \dots, \eta_n)$. Hint: Solve the system when $n = 1, 2, 3$ first. Do you see a pattern emerging?

3. Show that if a solution of $y'' + ty' + (1 + t^2)y^2 = 0$ touches the t axis at some point t_0 , then it is identically zero.