MATH 557, Fall 2007, Homework set # 11, Due: December 17, 2007

1. For the so-called Lienard equation $y''+g(y)y'+f(y)=0$ assume that $f(0)=0$ so that $y(t)=0$ is a solution. Find conditions on f and g which guarantee that the zero solution is asymptotically stable.

2. Work problem 11 on page 227 of the text.

3. Consider the system

$$
\begin{aligned}
x_1' &= \epsilon x_1 + x_2 - x_1(x_1^2 + x_2^2) \\
x_2' &= -x + \epsilon x_2 - x_2(x_1^2 + x_2^2)
\end{aligned}
$$

where ϵ is a parameter.

a) Introduce polar coordinates (r, θ) in the phase plane ($x_1 = r\cos\theta, x_2 = r\sin\theta$) and find the system satisfied by $(r(t), \theta(t))$.

b) Show that the system has a periodic solution for $\epsilon > 0$. (The appearance of a periodic solution as the parameter ϵ becomes positive is called a *Hopf bifurcation.*)

c) Show that the origin is a stable spiral point for $\epsilon < 0$ and an unstable spiral point for $\epsilon > 0$.

d) If $\epsilon > 0$, describe orbits which pass through points close to the origin.