You may consult with other human beings on these problems
Due date: December 17, 2008

Each problem is worth 10 points unless otherwise stated.

1. Prove that a closed set is nowhere dense if and only if it contains no open set. Prove that E is nowhere dense if and only if for every open set $O$ there is a nonempty ball in $O - E$.

2. Prove that if $A, B$ are closed disjoint subsets of a metric space and one of them, (say $A$), is compact, then $\rho(A, B) > 0$.

3. If $A, B$ are nowhere dense in $X$, then so is $A \cup B$. If $A$ is nowhere dense in $X$ and $B$ is nowhere dense in $Y$, then $A \times B$ is nowhere dense in $X \times Y$.

4. If $X, Y$ are first category, so is $X \times Y$. Prove that if $\{E_n\}$ is a sequence of first category sets, so is their union.

5. Definition: A point $x \in X$ is isolated if it is not a limit point of $X$. Prove that a space of the first category has no isolated points. Prove that a complete, countable metric space must have isolated points.

6. Show that on $[0, 1]$, there is a nowhere dense closed set having Lebesgue measure $1 - 1/n$. Construct a first category set in $[0, 1]$ with Lebesgue measure one.

7. 25 pts. Work problem 38, p 161 of the text.