

Math 515
Real Analysis
Problem Set 8

You may consult with other human beings on these problems

Due date: December 1, 2008

Each problem is worth 10 points unless otherwise stated.

1. Show that the completion of a separable metric space is separable. Show that the Cartesian product of two separable metric spaces is separable and conversely.
2. Show that the space of bounded sequences of real numbers (functions on $X = \{1, 2, 3, \dots\}$) is not separable.
3. Show that $L^1(0, 1)$ is separable and $L^\infty(0, 1)$ is not separable.
4. Let (X, ρ) be a metric space and $A \subset X$. For each $x \in X$ define, $\rho(x, \overline{A}) = \inf\{\rho(x, a) | a \in A\}$. Show that this is a continuous function of x and that it vanishes if and only if $x \in \overline{A}$.
5. Work problem 10, page 145 of the text.
6. If (X, ρ) is a metric space, then so is $(X, \rho/(1 + \rho))$.