1. 20 pts. Show that, in general, $f'(x)$ exists does not imply that $V'(x)$ exists where $V(x) = V([a, x])$ is the total variation of $f(\cdot)$ on $[a, x]$. Hint: consider, for $1 < \alpha < 2$, the function

$$f(x) = \begin{cases} x^2 \sin(x^{-\alpha}) & \text{for } 0 < x \leq 1 \\ 0 & \text{for } x = 0. \end{cases}$$

Show that $f'(0)$ exists but that $V'(0)$ does. (Consider only right hand derivatives at zero.) Hint: Consider $(f(h) - f(0))/h$ for the first part. For the second part let $x_n = (n\pi)^{-1/\alpha}$ and show that $V([x_{n+1}, x_n]) \geq c(n + 1)^{-2/\alpha}$. ($C, c$ are just computable constants.)

2. State and prove a version of Hölder’s inequality when $0 < p < 1$.


4. Prove that $l^p$ (sequence space) is complete if $1 \leq p < \infty$.

5. Work problem 16, p 126 in the text.