

Math 515  
Real Analysis  
Problem Set 7

You may consult with other human beings on these problems

Due date: November 12, 2008

1. 20 pts. Show that, in general,  $f'(x)$  exists does not imply that  $V'(x)$  exists where  $V(x) = V([a, x])$  is the total variation of  $f(\cdot)$  on  $[a, x]$ . Hint: consider, for  $1 < \alpha < 2$ , the function

$$f(x) = \begin{cases} x^2 \sin(x^{-\alpha}) & \text{for } 0 < x \leq 1 \\ 0 & \text{for } x = 0. \end{cases}$$

Show that  $f'(0)$  exists but that  $V'(0)$  does. (Consider only right hand derivatives at zero.) Hint: Consider  $(f(h) - f(0))/h$  for the first part. For the second part let  $x_n = (n\pi)^{-1/\alpha}$  and show that  $V([x_{n+1}, x_n]) \geq c(n+1)^{-2/\alpha}$ . ( $C, c$  are just computable constants.)

2. State and prove a version of Hölder's inequality when  $0 < p < 1$ .
3. Work problem 7, p122 in the text.
4. Prove that  $l^p$  (sequence space) is complete if  $1 \leq p < \infty$ .
5. Work problem 16, p 126 in the text.