

Math 515
Real Analysis
Problem Set 6

You may consult with other human beings on these problems

Due date: October 31, 2008

Each problem is worth 10 points unless otherwise stated.

1. 10pts. Show that the dominated convergence theorem follows from Fatou's theorem. (Hint: consider $g_n = 2g - |f_n - f|$.)

2. 10pts. Suppose $a < b$ and $a' < b'$. Let

$$f(x) = x \cdot \begin{cases} a \sin^2\left(\frac{1}{x}\right) + b \cos^2\left(\frac{1}{x}\right) & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ a' \sin^2\left(\frac{1}{x}\right) + b' \cos^2\left(\frac{1}{x}\right) & \text{if } x < 0. \end{cases}$$

Find the derivatives $D^+f(0)$, $D_+f(0)$, $D^-f(0)$, $D_-f(0)$.

3. 10pts. Show that

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is continuous on $[-1, 1]$ but is not of bounded variation there. Is

$$f(x) = \begin{cases} |x|^{3/2} \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

of bounded variation? Why or why not?

4. 10pts. Prove that a function of bounded variation can have at most countably many discontinuities.

5. 10pts. Prove the claim in class that $T_a^x(f)$ is increasing on $[a, b]$ if f is of bounded variation there.