Math 515

Real Analysis

Problem Set 4

You may not consult with other human beings on these problems

Due date: October 10, 2008

Each graded problem is worth 10 points.

1. Prove Egoroff's Theorm. If $\{f_n\}$ is a sequence of measurable functions that converges a.e. on a measurable set E, then for every $\eta > 0$ theer is $A \subset E$ with $m(A) < \eta$ such that the sequence converges uniformly on $E - A$.

2. Show that if $f(\cdot)$ is measurable, so is $|f(\cdot)|$. Is the converse true, why or why not?

3. Derive Chebyshev's inequality

$$m(\{x||f(x)| \geq n\}) \leq \frac{\int |f(x)|\, dm(x)}{n}.$$

4. Suppose $f \geq 0$ is measurable on a measurable set E and $\int_E f dx = 0$. Prove that $f = 0$ almost everywhere on E.

5. Suppose that f is integrable on a set E and $f_n \to f$ a. e. Assume that the $f_n \geq 0$ almost everywhere. Show that $\int |f_n(x) - f(x)|\, dm(x) \to 0$ if and only if $\int_E f_n(x)\, dm(x) \to \int_E f(x)\, dm(x)$. Hint: Apply Fatou's lemma with $g_n = |f| + |f_n| - |f - f_n|$. Show that if the hypothesis that the f_n are nonnegative is omitted, the reverse implication is false. Hint: Consider odd functions.