

Math 515
Real Analysis
Problem Set 3

Due date: September 30, 2005

Consultation with other individuals is permitted for this problem set.

Each graded problem is worth 10 points.

1. 5 pts. Show that the Cantor set has measure zero.
2. 10 pts. Fix some number $\alpha \in (0, 1)$. Construct a set similar to the Cantor set by removing at each stage, from the middle third of each interval at the n th stage, an open interval of length $\alpha/3^n$. Call the remaining set F and show that it is closed, its complement is dense in $[0, 1]$ but $m(F) = 1 - \alpha$.
3. 20 pts. Prove proposition 15 on page 63.
4. 20 pts. Show that outer measure is not countably additive. Hint: You will need the Axiom of choice for this.