

Math 515
Real Analysis
Problem Set 2

Due date: September 17, 2008

Consultation with other class members is encouraged for this problem set.

1. 15 pts. Show that the Cantor ternary set C is a closed, nowhere dense set of outer measure zero. (Recall that this set is constructed as a subset of $[0, 1]$ by first removing the open middle third of that interval, then removing the open middle third of the remaining two closed interval and so on. Show that $x \in C$ if and only if it can be represented as a sequence consisting of zeros and twos in base three. ($1.0 = .222\dots$, $1/3 = 0.02222\dots$). Show that this set has the same cardinality as the real line. (Hint: Identify each element with a binary sequence.) (A set is nowhere dense if the closure of the complementary set is dense.)

2. 10 pts. For any pair of bounded sequences $\{x_n\}$, $\{y_n\}$ show that $\liminf x_n + \limsup y_n \leq \limsup(x_n + y_n) \leq \limsup x_n + \limsup y_n$. What happens both sequences are unbounded?

3. 18 pts. Work problem 50 (c-h inclusive) on page 51.

4. 10 pts. Work problem 52 on page 53. Hint: Use part c of problem 50 and some rules about inverse images (problem 17, p 16 that you do not have to prove).

5. 12 pts. Work problems 53 and 54 on page 53. Again you need to use the definitions to show this.