Math 515

Real Analysis

Problem Set 1

Due date: September 5, 2008 - electronic submission of pdf files only please.

Consultation with other individuals is not permitted for this problem set.

1. (a) 5 pts. Define precisely (by consulting other texts if you need to do so) what is meant by the statement "$f(x)$ is a piecewise continuous function." You may assume the domain of this function is the closed, bounded interval $[a, b]$ and that it is real valued. (b) 10 pts. Prove that a piecewise continuous function on such an interval is Riemann integrable. (You may assume the result when $f(x)$ is continuous.)

2. 15 pts. Let, for $x \in [0, 1]$,

$$f(x) = \begin{cases} 1 & \text{if } x = \dfrac{p}{q} \\ 0 & x \text{ is irrational.} \end{cases}$$

Show carefully that $f(x)$ is not Riemann integrable.

3. 15 pts. Let, for $x \in [0, 1]$,

$$f(x) = \begin{cases} \dfrac{1}{q} & \text{if } x = \dfrac{p}{q} \\ 0 & x \text{ is irrational.} \end{cases}$$

Show carefully that $f(x)$ is Riemann integrable and give the value of the integral.

4. 15 pts. Let $\prec$ be a partial order on X. Show that there is a unique strict partial order $<$ and a unique reflexive partial order $\leq$ on X such that if $x \neq y$ then $(x, y) \in \prec \Leftrightarrow (x, y) \in < \Leftrightarrow (x, y) \in \leq$.

5. 15 pts. Give an example of a partially ordered set that has a unique minimal element but no smallest element.