“Introduction to Robotics ”

Dr. Greg R. Luecke
Associate Professor
Mechanical Engineering
Iowa State University

©2016 Dr. Greg R. Luecke
Plans for the semester:

Positions and orientations in space

FIGURE 1.5: Coordinate systems or “frames” are attached to the manipulator and to objects in the environment.

Plans for the semester:

Robot “forward kinematics”

Plans for the semester:

Robot “inverse kinematics”

Plans for the semester:

Velocity and forces-The Jacobian

FIGURE 1.8: The geometrical relationship between joint rates and velocity of the end-effector can be described in a matrix called the Jacobian.

Plans for the semester:

Robot motion with dynamics

**FIGURE 1.10:** The relationship between the torques applied by the actuators and the resulting motion of the manipulator is embodied in the dynamic equations of motion.

Plans for the semester:

Trajectory generation

![Diagram showing a robotic arm with joints labeled \( \theta_1, \theta_2, \theta_3 \) and points A and B.]

**FIGURE 1.11:** In order to move the end-effector through space from point A to point B, we must compute a trajectory for each joint to follow.

Plans for the semester:

Mechanical design considerations

FIGURE 1.12: The design of a mechanical manipulator must address issues of actuator choice, location, transmission system, structural stiffness, sensor location, and more.

Plans for the semester:

Control systems

Plans for the semester:

Constrained motion and forces control

*FIGURE 1.14: In order for a manipulator to slide across a surface while applying a constant force, a hybrid position–force control system must be used.*

Plans for the semester:

Some real robot motion considerations - Teach and Learn

Plans for the semester:

Concepts for advanced robot planning

Today:
Introduction, spatial descriptions, mappings, operators

Position

\[ \mathbf{A} \mathbf{P} \]

Describes the point \( P \) relative to the fixed coordinate system.

\[ \mathbf{A} \mathbf{P} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \]
Translation:

\[ A_\mathbf{P} = A_\mathbf{P}_{BORG} + B_\mathbf{P} \]

Describes the point \( \mathbf{P} \) relative to the new coordinate system.
Orientation: Often there is a convenient coordinate frame for feature description.

Describes the point “P” relative to the fixed coordinate system “B”.

\[
B P = \begin{pmatrix}
  p_x \\
  p_y \\
  p_z
\end{pmatrix}
\]
Orientation

Frame \{A\} is fixed in space, with convenient mapping for measurement.

Frame \{B\} is fixed to the object, and is associated with a datum on the object.
Orientation

How do we describe this vector? In frame \{A\}, we use the unit vectors: $x_A, y_A, z_A$
Orientation

Using frame \{B\} the unit vectors are: \(x_B, y_B, z_B\).
Unit vectors in \{B\} are:

\[
\hat{x}_B = 1 \hat{i}_B + 0 \hat{j}_B + 0 \hat{k}_B
\]

\[
B \hat{x}_B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}
\]
Unit vectors in \{A\} are:

\[
\hat{x}_B^A = \cos(\alpha) \hat{i}_A + \sin(\alpha) \hat{j}_A + 0 \hat{k}_A
\]

\[
\begin{align*}
\hat{x}_B \cdot \hat{x}_A &= \cos(\alpha) \\
\hat{x}_B \cdot \hat{y}_A &= \sin(\alpha) \\
\hat{x}_B \cdot \hat{z}_A &= 0
\end{align*}
\]

\[
\begin{bmatrix}
\hat{x}_B \cdot \hat{x}_A \\
\hat{x}_B \cdot \hat{y}_A \\
\hat{x}_B \cdot \hat{z}_A
\end{bmatrix} = 
\begin{bmatrix}
\cos(\alpha) \\
\sin(\alpha) \\
0
\end{bmatrix}
\]
Unit vectors in \{A\} are:

$$\hat{y}_B^A = -\sin(\alpha) \hat{i}_A + \cos(\alpha) \hat{j}_A + 0 \hat{k}_A$$

$$\hat{y}_B \cdot \hat{x}_A$$

$$\hat{y}_B \cdot \hat{y}_A$$

$$\hat{y}_B \cdot \hat{z}_A$$

$$\{B\} \rightarrow \{A\}$$

$$\hat{x}_A$$

$$\alpha$$

$$P$$
Unit vectors in {A} are:

\[
A \hat{Z}_B = 0 \hat{i}_A + 0 \hat{j}_A + 1 \hat{k}_A
\]

\[
\hat{z}_B \cdot \hat{x}_A \quad \hat{z}_B \cdot \hat{y}_A \quad \hat{z}_B \cdot \hat{z}_A
\]
Find vector "$\mathbf{P}_B$", but describe using unit vectors in $\{A\}$:

$$
\begin{align*}
\hat{A} \mathbf{P}_B &= \begin{bmatrix}
\hat{A} P_{Bx} \\
\hat{A} P_{By} \\
\hat{A} P_{Bz}
\end{bmatrix} = \\
&= \begin{bmatrix}
(\hat{x}_B \cdot \hat{x}_A) & (\hat{y}_B \cdot \hat{x}_A) & (\hat{z}_B \cdot \hat{x}_A) \\
(\hat{x}_B \cdot \hat{y}_A) & (\hat{y}_B \cdot \hat{y}_A) & (\hat{z}_B \cdot \hat{y}_A) \\
(\hat{x}_B \cdot \hat{z}_A) & (\hat{y}_B \cdot \hat{z}_A) & (\hat{z}_B \cdot \hat{z}_A)
\end{bmatrix}
\begin{bmatrix}
\hat{B} P_{Bx} \\
\hat{B} P_{By} \\
\hat{B} P_{Bz}
\end{bmatrix}
= \hat{A} R \hat{B} \mathbf{P}_B
\end{align*}
$$

"direction cosines"

$$
\begin{bmatrix}
(\hat{x}_B \cdot \hat{x}_A) & (\hat{y}_B \cdot \hat{x}_A) & (\hat{z}_B \cdot \hat{x}_A) \\
(\hat{x}_B \cdot \hat{y}_A) & (\hat{y}_B \cdot \hat{y}_A) & (\hat{z}_B \cdot \hat{y}_A) \\
(\hat{x}_B \cdot \hat{z}_A) & (\hat{y}_B \cdot \hat{z}_A) & (\hat{z}_B \cdot \hat{z}_A)
\end{bmatrix}
$$

$$
\hat{B} R = \begin{bmatrix}
\hat{A} \hat{x}_B \\
\hat{A} \hat{y}_B \\
\hat{A} \hat{z}_B
\end{bmatrix}
$$
The rotation from \{A\} to \{B\}:

\[
\begin{array}{ccc}
A_B R &=& \begin{bmatrix}
(x_B \cdot \hat{x}_A) & (y_B \cdot \hat{x}_A) & (z_B \cdot \hat{x}_A) \\
(x_B \cdot \hat{y}_A) & (y_B \cdot \hat{y}_A) & (z_B \cdot \hat{y}_A) \\
(x_B \cdot \hat{z}_A) & (y_B \cdot \hat{z}_A) & (z_B \cdot \hat{z}_A)
\end{bmatrix} \\
&=& \begin{bmatrix}
A_\hat{x}_B & A_\hat{y}_B & A_\hat{z}_B
\end{bmatrix} \begin{bmatrix}
\cos(\alpha) & -\sin(\alpha) & 0 \\
\sin(\alpha) & \cos(\alpha) & 0 \\
0 & 0 & 1
\end{bmatrix}
\end{array}
\]

\[
A_\hat{x}_B = B_R B_\hat{x}_B = \begin{bmatrix}
\cos(\alpha) & -\sin(\alpha) & 0 \\
\sin(\alpha) & \cos(\alpha) & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}1 \\0 \\0\end{bmatrix} = \begin{bmatrix}\cos(\alpha) \\ \sin(\alpha) \\ 0\end{bmatrix}
\]

\[
A_\hat{y}_B = B_R B_\hat{y}_B = \begin{bmatrix}
\cos(\alpha) & -\sin(\alpha) & 0 \\
\sin(\alpha) & \cos(\alpha) & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}0 \\1 \\0\end{bmatrix} = \begin{bmatrix}-\sin(\alpha) \\ \cos(\alpha) \\ 0\end{bmatrix}
\]
{A} Unit vectors in {B} are:

\[ B \hat{x}_A = \cos(\alpha)\hat{x}_B + -\sin(\alpha)\hat{y}_B + 0 * \hat{z}_B \]
Unit vectors in $\{B\}$ are:

$$\begin{align*}
\mathbf{y}_A^B &= \sin(\alpha) \mathbf{x}_B + \cos(\alpha) \mathbf{y}_B + 0 \cdot \mathbf{z}_B \\
\end{align*}$$
Suppose we look at it another way:

\[
\begin{pmatrix}
A P_{Bx} \\
A P_{By} \\
A P_{Bz}
\end{pmatrix} =
\begin{pmatrix}
(x_B \cdot \hat{x}_A) & (y_B \cdot \hat{x}_A) & (z_B \cdot \hat{x}_A) \\
(x_B \cdot \hat{y}_A) & (y_B \cdot \hat{y}_A) & (z_B \cdot \hat{y}_A) \\
(x_B \cdot \hat{z}_A) & (y_B \cdot \hat{z}_A) & (z_B \cdot \hat{z}_A)
\end{pmatrix}
\begin{pmatrix}
A P_{Bx} \\
A P_{By} \\
A P_{Bz}
\end{pmatrix} =
\begin{pmatrix}
A R^B P_B
\end{pmatrix}
\]

Note: \( \hat{y}_A \cdot \hat{x}_B = \hat{x}_B \cdot \hat{y}_A \), etc...

\[
\begin{pmatrix}
B P_{Ax} \\
B P_{Ay} \\
B P_{Az}
\end{pmatrix} =
\begin{pmatrix}
(x_A \cdot \hat{x}_B) & (y_A \cdot \hat{x}_B) & (z_A \cdot \hat{x}_B) \\
(x_A \cdot \hat{y}_B) & (y_A \cdot \hat{y}_B) & (z_A \cdot \hat{y}_B) \\
(x_A \cdot \hat{z}_B) & (y_A \cdot \hat{z}_B) & (z_A \cdot \hat{z}_B)
\end{pmatrix}
\begin{pmatrix}
A P_{Ax} \\
A P_{Ay} \\
A P_{Az}
\end{pmatrix} =
\begin{pmatrix}
A R^A P_A
\end{pmatrix}
\]

\[
A_R = B_R^T = B_R^{-1}
\]

Notice:

\[
B_R = A_R^T = B_R^{-1}
\]
Notice that the rotation matrix is made up of elements of each unit vector:

\[
{^A_R^B} = \begin{bmatrix}
{^A X_B} & {^A Y_B} & {^A Z_B}
\end{bmatrix} = 
\begin{bmatrix}
{^B X_A} & {^B Y_A} & {^B Z_A}
\end{bmatrix}
\]
Any vector can be described in either frame using this rotation matrix:

\[
B P = \begin{bmatrix}
B p_x \\
B p_y \\
B p_z
\end{bmatrix}
\]

\[
A P = _B^A R B P = \begin{bmatrix}
A p_x \\
A p_y \\
A p_z
\end{bmatrix}
\]
Example:
Rotation around the z-axis

\[ \hat{x}_B \cdot \hat{x}_A = \cos(\alpha) \]

\[ \hat{x}_B \cdot \hat{y}_A = \cos(90 - \alpha) = \sin(\alpha) \]

\[ \hat{x}_B \cdot \hat{z}_A = 0 \]

\[ \begin{bmatrix}
\cos(\alpha) & -\sin(\alpha) & 0 \\
\sin(\alpha) & \cos(\alpha) & 0 \\
0 & 0 & 1
\end{bmatrix} \]

\[ A_B R = \begin{bmatrix}
\cos(\alpha) & -\sin(\alpha) & 0 \\
\sin(\alpha) & \cos(\alpha) & 0 \\
0 & 0 & 1
\end{bmatrix} \]
\{B\} Unit vectors in \{A\}:

\[ A \hat{x}_B = B R^B \hat{x}_B \]

\[ B \hat{x}_A = \begin{bmatrix} \cos(\alpha) \\ -\sin(\alpha) \\ 0 \end{bmatrix} \]
Unit vectors in \{A\} are:

\[
A \hat{x}_B = \underbrace{\cos(\alpha) i_A}_{\hat{x}_B \bullet \hat{x}_A} + \underbrace{\sin(\alpha) j_A}_{\hat{x}_B \bullet \hat{y}_A} + \underbrace{0 \ast k_A}_{\hat{x}_B \bullet \hat{z}_A}
\]

\[
\begin{align*}
\hat{x}_B \bullet \hat{x}_A &= \cos(\alpha) \\
\hat{x}_B \bullet \hat{y}_A &= \sin(\alpha) \\
\hat{x}_B \bullet \hat{z}_A &= 0
\end{align*}
\]

\[
A \hat{x}_B = \begin{bmatrix} \hat{x}_B \bullet \hat{x}_A \\ \hat{x}_B \bullet \hat{y}_A \\ \hat{x}_B \bullet \hat{z}_A \end{bmatrix} = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \\ 0 \end{bmatrix}
\]