Multi-Round Sensor Deployment for Guaranteed Barrier Coverage

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***Abstract*—This report documents the omitted calculations and derivations in our submitted conference paper (Section V: Practical Considerations).**

The Pilot Deployment Solution

Now we present a more efficient solution by introducing an additional pilot round prior to the two rounds of sensor deployment. The basic idea is to use the residence points of the sensors deployed in the pilot round to estimate σ_{real}, which is then used to guide the next two rounds of sensor deployment.

Let N_{pilot} and I_{pilot} denote the number of sensors deployed in the pilot round and the deployment interval. The deployment could start from either t_{left} or t_{right}. Recall that, in order to guarantee coverage of the left (right) boundary of the barrier, t_{left} (t_{right}) should be at a distance of $(R_s - 3\sigma_{\max})$ from the left (right) end of the deployment line. After the pilot round, the residence points of deployed sensors are collected and used to estimate σ_{real} as follows. The sample variance of the deviation of the residence points along X-axis and Y-axis with respect to the corresponding deployment points is calculated as follows:

$$S = \frac{\sum_{i=1}^{N_{\text{pilot}}} (x'_i - x_i)^2 + \sum_{i=1}^{N_{\text{pilot}}} (y'_i)^2}{2N_{\text{pilot}} - 1}, \tag{1}$$

based on which we propose the following estimator for σ_{real}:

$$\hat{\sigma}_{\text{real}} = \begin{cases} \sigma_{\min}, & \text{if } S \leqslant \sigma_{\min}^2, \\ \sqrt{S}, & \text{if } \sigma_{\min}^2 < S < \sigma_{\max}^2, \\ \sigma_{\max}, & \text{if } S \geqslant \sigma_{\max}^2. \end{cases} \tag{2}$$

This estimator makes sense because the truncated two-dimensional Gaussian distribution (symmetrically at 3σ) is very similar to the non-truncated version. Therefore, the deviations along X-axis and Y-axis, i.e., $(x'_i - x_i)$ and y'_i, can be treated as two sets of independent samples to collectively contribute to the estimation of σ_{real}. Since the sample variance of the Gaussian distribution follows the Gamma distribution [1], [2] with parameters $(\frac{2N_{\text{pilot}}-1}{2}, \frac{2\sigma_{\text{real}}^2}{2N_{\text{pilot}}-1})$, by performing a function transformation, the pdf of the sample standard deviation (square root of S) can be calculated as follows:

$$f_{\sqrt{S}}(s|\sigma_{\text{real}}) = f_{\Gamma}\left(s^2; \frac{2N_{\text{pilot}} - 1}{2}, \frac{2\sigma_{\text{real}}^2}{2N_{\text{pilot}} - 1}\right) \cdot 2s, \tag{3}$$

where the expression of function $f_{\Gamma}(\cdot)$ can be found in [1]. Due to the truncation (between $\sigma_{\min}^2$ and $\sigma_{\max}^2$) in the estimator $\hat{\sigma}_{\text{real}}$, the conditional pdf of $\hat{\sigma}_{\text{real}}$ is approximately:

$$\begin{aligned} f_{\hat{\sigma}_{\text{real}}}(\hat{\sigma}|\sigma_{\text{real}}) = {} & B_1 \cdot \delta(\hat{\sigma} - \sigma_{\min}) + B_2 \cdot \delta(\hat{\sigma} - \sigma_{\max}) \\ & + I_{\hat{\sigma}} \cdot f_{\Gamma}\left(\hat{\sigma}^2; \frac{2N_{\text{pilot}} - 1}{2}, \frac{2\sigma_{\text{real}}^2}{2N_{\text{pilot}} - 1}\right) \cdot 2\hat{\sigma}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} B_1 &= \int_0^{\sigma_{\min}} f_{\Gamma}\left(\hat{\sigma}^2; \frac{2N_{\text{pilot}} - 1}{2}, \frac{2\sigma_{\text{real}}^2}{2N_{\text{pilot}} - 1}\right) \cdot 2\hat{\sigma}\, d\hat{\sigma}, \\ B_2 &= \int_{\sigma_{\max}}^{\infty} f_{\Gamma}\left(\hat{\sigma}^2; \frac{2N_{\text{pilot}} - 1}{2}, \frac{2\sigma_{\text{real}}^2}{2N_{\text{pilot}} - 1}\right) \cdot 2\hat{\sigma}\, d\hat{\sigma}, \\ I_{\hat{\sigma}} &= \begin{cases} 1, & \text{if } \sigma_{\min} \leqslant \hat{\sigma} \leqslant \sigma_{\max}, \\ 0, & \text{otherwise}, \end{cases} \end{aligned} \tag{5}$$

$f_{\Gamma}(\cdot)$ and $\delta(\cdot)$ are the Gamma distribution function and the unit impulse function, respectively. Then, the next two rounds of sensor deployment are planned as follows:

- For each coverage gap (with a gap distance of h) generated after the pilot round, deploy $N^*_{\hat{\sigma}_{\text{real}}}(h, 3, 2)$ sensors in the first round, where $N^*_{\hat{\sigma}_{\text{real}}}(h, 3, 2)$ is obtained based on the assumption that $\sigma_{\text{real}} = \hat{\sigma}_{\text{real}}$;
- For each coverage gap of size h generated after the first round, deploy $\left(\left\lceil \frac{h}{2(R_s - 3\sigma_{\max})} \right\rceil + 1\right)$ sensors in the second round to guarantee coverage of the gap.

Finally, we can find the optimal $<N^*_{\text{pilot}}, I^*_{\text{pilot}}>$ that minimizes the maximum number of extra sensors deployed when the pilot deployment solution is used:

$$\begin{aligned} & < N^*_{\text{pilot}}, I^*_{\text{pilot}} >= \arg \min_{<N_{\text{pilot}}, I_{\text{pilot}}>} \max_{\sigma_{\text{real}}} \\ & \left[\int_0^{\infty} \bar{\mathcal{N}}^p_{\text{total}}(h_\ell, N_{\text{pilot}}, I_{\text{pilot}}, \sigma_{\text{real}}, \hat{\sigma}) \cdot f_{\hat{\sigma}_{\text{real}}}(\hat{\sigma}|\sigma_{\text{real}})\, d\hat{\sigma} - \bar{\mathcal{N}}^*_{\text{total}}(h_\ell, 2) \right], \end{aligned} \tag{6}$$

where $\bar{\mathcal{N}}^p_{\text{total}}(h_\ell, N_{\text{pilot}}, I_{\text{pilot}}, \sigma_{\text{real}}, \hat{\sigma})$ is the expected total number of sensors needed to cover the barrier by following the above pilot deployment solution, whose analysis is similar to that of $\bar{\mathcal{N}}_{\text{total}}$ in Section IV.

REFERENCES

[1] "http://en.wikipedia.org/wiki/gamma_distribution."
[2] "http://en.wikipedia.org/wiki/normal_distribution#estimation_of_parameters."