Barrier Information Coverage with Wireless Sensors

***Abstract*—In this short report, we will include the omitted theorem proofs in our recent submission to INFOCOM 2009.**

Omitted Theorem Proofs

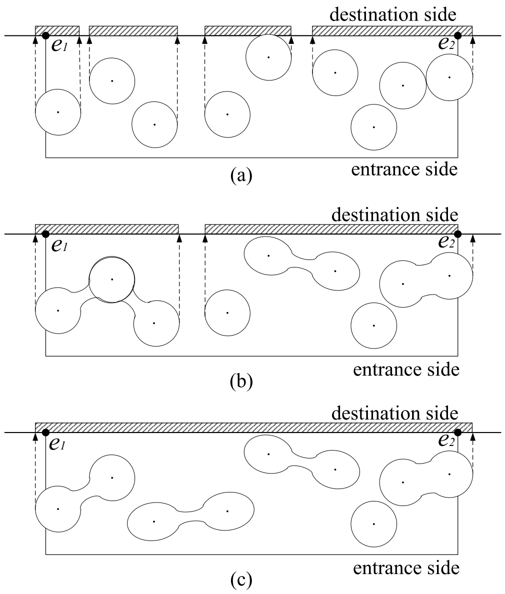

Fig. 1. Illustration of barrier information coverage. The destination side of the barrier is the line segment $\overline{e_1 e_2}$. (a) Without sensor collaborations, the barrier is not covered. (b) Without well-planned sensor collaborations, the barrier is not covered. (c) Barrier is information-covered with well-planned sensor collaborations.

As mentioned in our submission, in the following theorems and proofs, the positions of two collaborating sensors (s_1 and s_2) are symmetrically located at the left and right side of the origin and along the X-axis, respectively. The distance between them is denoted by L.

Theorem 1: A barrier is information-covered if and only if the barrier projections of the coverage regions of active sensors can cover the entire destination side of the barrier (as shown in Fig. 1).

Proof: Our proof contains two parts. (1) *Barrier information coverage ⇒ line coverage*: If there exists a point t on the barrier's destination side, which is not within the projection of any sensor's coverage region, then there must exist a perpendicular intruding path ℓ (intersecting with the barrier's destination side at point t) that is not covered by any sensor. Thus, we prove "⇒". (2) *Barrier information coverage ⇐ line coverage*: If the barrier is not information covered, then there exists an intruding path ℓ that is not covered by any sensor. Suppose that ℓ intersects with the barrier's destination side at t. Thus, t is not contained in the projection of any sensor's coverage region. So we prove "⇐". ■

Theorem 2: The coverage region of a virtual sensor can be either connected or disconnected (with two disconnected symmetric subregions) according to the distance L between two collaborating sensors.

Proof: We divide the proof into two cases according to L.

- When $L \leqslant L_{\text{critical}}$, we know that the origin in the coordinate system should be covered according to the sensing intensity of the virtual sensor at the origin. Also, due to the symmetrical positions of two sensors, the boundary of any sensor's coverage region should be symmetrical distributed according to both the X-axis and Y-axis, i.e., any connected coverage region should exist in all of the four sections of the coordinate system. Actually, this implies that any connected coverage region should contain the origin. Because, if the origin is not covered, any point t along the Y-axis will not be covered, either:

$$I(d(t, s_1), d(t, s_2)) \leqslant I(d(o, s_1), d(o, s_2)) < I_{\text{threshold}}, \quad (1)$$

where the point o is the origin. Hence the coverage region existing in four sections will be disconnected, which contradicts with the condition. Thus, when $L < L_{\text{critical}}$, all coverage regions should contain the origin, and, hence, are all connected as one region.
- When $L > L_{\text{critical}}$, we know there are at least two coverage subregions denoted by C_1 and C_2 (containing s_1 and s_2 respectively) on the origin's left and right. Note that the origin is not covered according to its sensing intensity. Now, we will prove there does not exist any other coverage subregion (C_3) that is connected with C_1 or C_2. Assume there exists a coverage subregion C_3, which is not connected with C_1 and C_2, and a point t within it. Let b denote the projection of point t on the X-aixs. Then, because

$$I(d(b, s_1), d(b, s_2)) \geqslant I(d(t, s_1), d(t, s_2)) \geqslant I_{\text{threshold}}, \quad (2)$$

the point b is also covered. In fact, it is easy to see that all the points along the segment bt are covered, too. So the point b is also within C_3. Using the similar proof for Lemma 3, we can show that any point on the segment bs_1 (or bs_2), if b is on the left (or right) side of Y-axis, should be covered, too. This contradicts with the condition that C_3 is not connected with C_1 (or C_2). Therefore, there only exist two coverage subregions C_1 and C_2 containing s_1 and s_2, respectively, when $L > L_{\text{critical}}$.

■

Theorem 3: The rightmost (or leftmost) point in a sensor's coverage region is always along the X-axis.

Proof: Suppose there is a point t within the coverage region (or subregion when $L > L_{\text{critical}}$), and its projection, point b, is on the X-axis. Because

$$I(d(b, s_1), d(b, s_2)) \geqslant I(d(t, s_1), d(t, s_2)) > I_{\text{threshold}}, \quad (3)$$

the point b on the X-axis is also covered. Since such result holds for any point t in the coverage region, the coverage region must have a common set with X-axis, which is a segment along X-axis. Let e denote the right end point of the segment. Suppose a line ℓ perpendicularly intersects with X-axis at the point e. Since we have

$$I(d(g, s_1), d(g, s_2)) < I(d(e, s_1), d(e, s_2)) = I_{\text{threshold}}, \quad (4)$$

where g is a point on the line ℓ except the point e. This means all the points along ℓ except the point e are not covered. Thus, we prove that the leftmost point e of a coverage region is along the X-axis. Similarly, we can prove that the rightmost point of a coverage region is also along the X-axis. ■