

Analytical Study of Sensor Network Lifetime under Connected- k -Coverage Constraint

Abstract

In this technical report, we provide the omitted theoretical analysis in our submitted journal version.

I. LOWER BOUNDS ON THE PROBABILITY OF P_{Cov} AND P_{Com}

In this section, we derive the lower bounds of P_{cov} and P_{com} , respectively.

A. Lower Bound on P_{cov}

LEMMA 1 *Let n points distributed independently and uniformly in a square field \mathcal{D} of unit area within \mathbb{R}^2 , then for sufficiently large n , these points form a stationary Poisson point process with density n .*

Lemma 1 is a well-known result and its proof is given by Hall in [17]. Let $\mathcal{P} \equiv \{\xi_i, i \geq 1\}$ denote the set of active sensors. It is shown in Lemma 2 that \mathcal{P} is also a stationary Poisson point process with density np for sufficiently large n .

LEMMA 2 *Let n points distributed independently and uniformly in a square field \mathcal{D} of unit area within \mathbb{R}^2 . Each point is marked independently as an active point with probability p , where $0 < p \leq 1$. Then the set of active points, $\mathcal{P} = \{\xi_i, i \geq 1\}$, is a stationary Poisson point process with density np for sufficiently large n .*

Proof: For M disjoint Borel sets $\{\mathcal{S}_m\}_{m=1}^M$, where $\mathcal{S}_m \in \mathcal{D}$ and $M \geq 1$, let x_m denote the number of active points ξ_i in \mathcal{S}_m , y_m the number of points (active or non-active) in \mathcal{S}_m , $\|\mathcal{S}_m\|$

the area of \mathcal{S}_m . For any $j \geq 0$, the limit of the probability of $x_m = j$, as n goes to infinity, is

$$\begin{aligned}
\lim_{n \rightarrow \infty} P(x_m = j) &= \lim_{n \rightarrow \infty} \sum_{i=j}^n P(x_m = j | y_m = i) \cdot P(y_m = i) \\
&= \lim_{n \rightarrow \infty} \sum_{i=j}^{\infty} P(x_m = j | y_m = i) \cdot P(y_m = i) \\
&= \sum_{i=j}^{\infty} P(x_m = j | y_m = i) \cdot \lim_{n \rightarrow \infty} P(y_m = i) \\
&= \sum_{i=j}^{\infty} \binom{i}{j} p^j (1-p)^{i-j} \cdot \frac{(n \|\mathcal{S}\|)^i}{i!} e^{-n \|\mathcal{S}_m\|} \\
&= \frac{(np \|\mathcal{S}_m\|)^j}{j!} e^{-np \|\mathcal{S}_m\|},
\end{aligned}$$

where the second equality is due to the fact that $P(x_m = j) \leq 1$, the third equality is the result of Monotone Convergence Theorem (MCT) [19], and the fourth equality is from the Lemma 1. Thus in the limit, the number of active points in \mathcal{S}_m is Poisson distributed with mean $np \|\mathcal{S}_m\|$.

Next, we prove by induction that x_m 's are independent random variables. The exchange of limit and summation used above will also be applied in the following proof, therefore all limits are omitted. According to the Lemma 1, all these points (active or non-active) form a stationary Poisson process, which means that y_m 's are independent Poisson random variables. All probability mass functions like $p(x_m = j_m)$ will be abbreviated as $p(x_m)$.

$$\begin{aligned}
p(x_1 | x_2) &= \sum_{y_1} p(x_1 | x_2, y_1) p(y_1 | x_2) \\
&= \sum_{y_1} p(x_1 | y_1) p(y_1 | x_2),
\end{aligned} \tag{1}$$

where the second equality is due to the fact that given y_i , x_i is independent of x_j and y_j for $i \neq j$. Since

$$\begin{aligned}
p(y_1 | x_2) &= \frac{p(y_1, x_2)}{p(x_2)} = \frac{\sum_{y_2} p(x_2, y_1, y_2)}{p(x_2)} \\
&= \frac{\sum_{y_2} p(x_2 | y_1, y_2) p(y_1) p(y_2)}{p(x_2)} \\
&= \frac{\sum_{y_2} p(x_2 | y_2) p(y_1) p(y_2)}{p(x_2)} \\
&= p(y_1),
\end{aligned} \tag{2}$$

the Eq. (1) can be written as

$$p(x_1|x_2) = p(x_1), \quad (3)$$

which proves that x_1 and x_2 are independent.

For $2 < m \leq M$, we have

$$\begin{aligned} & p(x_m|x_1, \dots, x_{m-1}) \\ &= \sum_{y_m} p(x_m|x_1, \dots, x_{m-1}, y_m)p(y_m|x_1, \dots, x_{m-1}) \\ &= \sum_{y_m} p(x_m|y_m)p(y_m|x_1, \dots, x_{m-1}), \end{aligned} \quad (4)$$

and

$$\begin{aligned} & p(y_m|x_1, \dots, x_{m-1}) \\ &= \frac{\sum_{y_{m-1}} p(x_1, \dots, x_{m-1}, y_m|y_{m-1})p(y_{m-1})}{p(x_1, \dots, x_{m-1})}. \end{aligned} \quad (5)$$

where

$$\begin{aligned} & p(x_1, \dots, x_{m-1}, y_m|y_{m-1})p(y_{m-1}) \\ &= p(x_{m-1}, y_m|y_{m-1}, x_1, \dots, x_{m-2})p(x_1, \dots, x_{m-2}|y_{m-1}) \\ &= p(x_{m-1}|y_{m-1})p(y_m|x_1, \dots, x_{m-2})p(x_1, \dots, x_{m-2}|y_{m-1}). \end{aligned} \quad (6)$$

Suppose $\{x_i\}_{i=1}^{m-1}$ are independent, it is easy to prove that

$$p(x_1, \dots, x_{m-2}|y_{m-1}) = p(x_1, \dots, x_{m-2}), \quad (7)$$

Plug (6) and (7) into (5), we obtain

$$\begin{aligned} & p(y_m|x_1, \dots, x_{m-1}) \\ &= \frac{1}{p(x_{m-1})} \sum_{y_{m-1}} p(y_m|x_1, \dots, x_{m-2})p(x_{m-1}|y_{m-1})p(y_{m-1}) \\ &= p(y_m|x_1, \dots, x_{m-2}). \end{aligned} \quad (8)$$

Recursively,

$$p(y_k|x_1, \dots, x_{m-1}) = p(y_m|x_1) = p(y_m), \quad (9)$$

where the second equality is due to (2). Next, plug (9) into (4) such that

$$p(x_m|x_1, \dots, x_{m-1}) = p(x_m). \quad (10)$$

Since $\{x_i\}_{i=1}^{m-1}$ are independent, thus $\{x_i\}_{i=1}^m$ are independent. By induction on m , we proved that x_m 's are independent random variables.

Based on the two results above, we conclude that the set \mathcal{P} of active points is a stationary Poisson process with density np for sufficiently large n . \square

Let S_i denote a random disc with radius $r_{s,i}$ centered at the origin of \mathbb{R}^2 , which is defined as $S_i \equiv \{x \in \mathbb{R}^2 : |x| \leq r_{s,i}\}$, where $r_{s,i}$ is the sensing radius of the i -th active sensor ξ_i . Here, we assume that all sensing radii are i.i.d random variables following an *unknown* distribution $F(r)$, with *known* mean r_0 and variance $r_0^2\sigma_s^2$, i.e., all S_i 's are distributed as S :

$$S \equiv \{x \in \mathbb{R}^2 : |x| \leq r, r \sim F(r)\}. \quad (11)$$

Then, the sensing disc (abbreviated as disc) centered at active sensor ξ_i can be defined as $D_i \equiv \xi_i + S_i = \{\xi_i + y : y \in S_i\}$. The set of $\{D_i, i \geq 1\}$ forms a stationary coverage process. For such a coverage process, Lemma 3 gives the distribution of the number of discs with certain properties.

LEMMA 3 *Let $\mathcal{Q} = \{\xi_i + S_i, i \geq 1\}$ denote a stationary coverage process, where $\{\xi_i\}$ is a stationary Poisson point process with density λ within \mathcal{D} , and S_i 's are distributed as S defined in (11). For a given deterministic condition C , let Y denote the number of discs in \mathcal{Q} that satisfy the condition C . Then, Y is Poisson-distributed with mean*

$$\mu = \lambda \cdot \mathbb{E}[\|\{x : I_C(x + S) = 1\}\|],$$

where $I_C(\cdot)$ is the indicator function of whether a disc satisfies the condition C or not, and $\|\cdot\|$ denotes the area.

Proof: Let N denote the number of points ξ_i 's within \mathcal{D} , then it is Poisson distributed with mean λ . Since all S_i 's are distributed as S and ξ_i 's are from a stationary Poisson process, conditional on N , the events that individual disc $(\xi_i + S_i)$ satisfies the deterministic condition

C are independent and have the same probability, defined as

$$p_0 \equiv P(I_C(\xi_i + S_i) = 1|N). \quad (12)$$

Since

$$Y = \sum_i I_C(\xi_i + S_i), \quad (13)$$

then

$$\begin{aligned} P(Y = y) &= \sum_{m=y}^{\infty} P(Y = y|N = m) \cdot P(N = m) \\ &= \sum_{m=y}^{\infty} \binom{m}{y} p_0^y (1 - p_0)^{m-y} \frac{\lambda^m e^{-\lambda}}{m!} \\ &= \frac{(\lambda p_0)^y e^{-\lambda p_0}}{y!}. \end{aligned}$$

By Fubini's theorem, we have

$$\begin{aligned} \mu \equiv E[Y] &= E\left[\sum_i I_C(\xi_i + S_i)\right] \\ &= \lambda E\left[\int_{\mathcal{D}} I_C(x + S) dx\right] \\ &= \lambda E\left[\|\{x : I_C(x + S) = 1\}\|\right]. \end{aligned}$$

Therefore, Y is Poisson distributed with mean μ . □

Let $Y(x)$ denote the number of active sensors that cover a point x , and $I_k(x)$ denote the indicator function of whether the point x is covered by at most $(k - 1)$ active sensors, i.e.,

$$I_k(x) = \begin{cases} 1, & \text{if } Y(x) < k, \\ 0, & \text{otherwise.} \end{cases}$$

Then, the expectation of Bernoulli random variable $I_k(x)$ is

$$E[I_k(x)] = P(x \text{ is at most } (k - 1)\text{-covered}) = P(Y(x) < k).$$

By Lemma 1, we know that $Y(x)$ is Poisson-distributed with mean

$$\mu = np \cdot E[\|\{x : (x + S) \cap \{x_0\} \neq \emptyset\}\|] = np \cdot E[\|x_0 - S\|] = np a_s,$$

where $a_s \equiv \mathbb{E}[\|S\|] = \pi r_0^2(1 + \sigma_s^2)$. Therefore,

$$\mathbb{E}[I_k(x)] = e^{-npa_s} \sum_{j=0}^{k-1} \frac{(npa_s)^j}{j!}. \quad (14)$$

K -coverage of the sensing field \mathcal{D} means that each point in \mathcal{D} should be covered by at least k active sensors. Next, we will give a lower bound of P_{Cov} for finite np . A Similar bound has been proved in [13] for the case of deterministic sensing radius model and non-sleeping sensor networks. Theorem 1 is a generalization of the results in [13] for the random sensing radius model.

THEOREM 1 For $n > 1$, $0 < p \leq 1$, and $a_s < 1$,

$$P_{cov} > P_{Cov}^L, \quad (15)$$

in which

$$P_{cov}^L = 1 - 2e^{-npa_s} \left(1 + (n^2 p^2 a'_s + 2np r_0) \sum_{i=0}^{k-1} \frac{(npa_s)^i}{i!} \right), \quad (16)$$

where $a'_s \equiv \pi r_0^2(1 + \sigma_s^2/2)$.

Proof: Observe that

$$P_{cov} = 1 - p_1 - p_2 - p_3,$$

where

$$p_1 = P(\text{no active sensors centered within } \mathcal{D}) = e^{-np} < e^{-npa_s}.$$

Here, we assume $a_s < 1$, meaning that, even for the random sensing radius model, the expected sensing area of one sensor will not cover the entire field \mathcal{D} .

$$\begin{aligned} p_2 &= P(\text{at least one disc centered within } \mathcal{D}, \text{ but none of the discs intersects with} \\ &\quad \text{any other disc, and none of the discs intersect the boundary of } \mathcal{D}) \\ &\leq P(\text{at least one disc centered within } \mathcal{D}) \cdot P(\text{a given disc intersects with no other discs}) \\ &= (1 - e^{-np}) \cdot e^{-np\pi\mathbb{E}[\pi(r_{s,1}+r_{s,2})^2]} = (1 - e^{-np}) \cdot e^{-2np\pi r_0^2(2+\sigma_s^2)} < e^{-npa_s}, \end{aligned}$$

where $r_{s,1}$ and $r_{s,2}$ are sensing radii of two active sensors, which are i.i.d with mean r_0 and variance $r_0^2\sigma_s^2$, and the second equality is due to Lemma 3.

$$\begin{aligned} p_3 &= P(\mathcal{D} \text{ is not } k\text{-covered, at least one disc centered within } \mathcal{D}, \text{ and at least} \\ &\quad \text{one disc intersects with another disc or the boundary of } \mathcal{D}). \end{aligned}$$

Therefore

$$P_{cov} > 1 - 2e^{-npa_s} - p_3. \quad (17)$$

Our next task is to derive an upper bound on p_3 .

Define a *crossing* to be either an intersection point of the boundaries of two discs or an intersection point of the boundary of an disc and the boundary of the field \mathcal{D} . A crossing is said to be k -covered if it is within at least k discs. It is proved in [8] that, field \mathcal{D} is k -covered if there exist crossings and every crossing is k -covered. Therefore, if \mathcal{D} is not k -covered, if one or more discs are centered within \mathcal{D} , and if there exist crossings in \mathcal{D} , then at least one of the discs has two or more crossings that are not k -covered. Thus

$$p_3 \leq P(M_k \geq 2) \leq E[M_k]/2, \quad (18)$$

where M_k denotes the number of crossings that are not k -covered.

Define L_1 and L_2 as the number of crossings created by two discs intersecting with each other, and the ones created by a disc intersecting the boundary of field \mathcal{D} . We first study the expected value of L_1 . The expected number of crossings created by a given active sensor ξ_1 with other active sensors is

$$E[2np \cdot \pi(r_{s,1} + r_{s,2})^2] = 8npa'_s,$$

where $a'_s \equiv \pi r_0^2(1 + \sigma_s^2/2)$, and the expected number of discs centered within \mathcal{D} is np . Therefore,

$$E[L_1] = np \cdot 8npa'_s/2 = 4n^2p^2a'_s.$$

If a disc intersects the edge of field \mathcal{D} , at most two crossings will be created; if a disc intersects the corner of field \mathcal{D} , at most four crossings will be created (due to the toroidal model assumption). Thus the expected value of L_2 is bounded by

$$E[L_2] \leq 8npr_0.$$

The probability that a given crossing is not k -covered is given by (14). Therefore,

$$E[M_k] = (E[L_1] + E[L_2])e^{-npa_s} \sum_{j=0}^{k-1} \frac{(npa_s)^j}{j!} \leq 4(n^2p^2a'_s + 2npr_0)e^{-npa_s} \sum_{j=0}^{k-1} \frac{(npa_s)^j}{j!}. \quad (19)$$

By (17), (18), and (19), we have

$$P_{cov} > P_{cov}^L \equiv 1 - 2e^{-npa_s} \left(1 + (n^2 p^2 a'_s + 2npr_0) \sum_{i=0}^{k-1} \frac{(npa_s)^i}{i!} \right).$$

This completes the proof. \square

B. Lower Bound on P_{com}

LEMMA 4 *Given the number of sensors deployed, the number of neighboring sensors that can communicate with a given sensor is Poisson-distributed with mean*

$$\mu = na_c, \quad \text{where } a_c = \pi \bar{\mathcal{L}}^2 (\sigma_c^2 + 1).$$

Proof: At first, we will calculate the probability (p_o) that an arbitrary sensor is a communication neighbor of a given sensor (denoted by s). Note that the distance between two communication neighbors must be shorter than \mathcal{L}_U . Suppose \mathcal{L} 's pdf and cdf are $f(\mathcal{L})$ and $F(\mathcal{L})$, respectively. The calculation is shown as follows:

$$\begin{aligned} p_o &= \int_0^{\mathcal{L}_U} (1 - F(\mathcal{L})) 2\pi \mathcal{L} d\mathcal{L} \\ &= \mathcal{L}_U^2 \pi - \int_0^{\mathcal{L}_U} F(\mathcal{L}) 2\pi \mathcal{L} d\mathcal{L} \\ &= \mathcal{L}_U^2 \pi - \mathcal{L}_U^2 \pi + \int_0^{\mathcal{L}_U} f(\mathcal{L}) \mathcal{L}^2 \pi d\mathcal{L} \\ &= \pi \sigma_c^2 \bar{\mathcal{L}}^2 + \pi \bar{\mathcal{L}}^2 = a_c. \end{aligned} \tag{20}$$

Secondly, similar with the proof of Lemma 3 in the technical report, we can prove that the number of communication neighbors of sensor s , denoted by Y follows Poisson distribution with the parameter na_c as below:

$$\begin{aligned} P(Y = y) &= \sum_{m=y}^{\infty} P(Y = y | N = m) \cdot P(N = m) \\ &= \sum_{m=y}^{\infty} \binom{m}{y} a_c^y (1 - a_c)^{m-y} \cdot \frac{n^m e^{-n}}{m!} \\ &= \frac{(na_c)^y e^{-na_c}}{y!}. \end{aligned} \tag{21}$$

Thus, we complete the proof. \square

THEOREM 2 *For $n > 1$, $0 < p \leq 1$, $a_s < 1$ and $a_c < 1$,*

$$P_{com} > P_{com}^L, \quad \text{in which} \tag{22}$$

$$P_{com}^L = \begin{cases} 1 - npe^{-npac} \cdot \min\{1, \frac{1}{2} \cdot (8npa'_s + 4n^2p^2a'_sa_s) \cdot e^{-\lambda_c} \sum_{j=0}^{k-1} \frac{(\lambda_c)^j}{j!}\} & \text{if } r_L > \mathcal{L}_U, \\ 1 - npe^{-npac} & \text{if } r_L \leq \mathcal{L}_U, \end{cases}$$

where $\lambda_c = np(a_s - \mathcal{L}_U^2\pi)(1 - (np)^2e^{-2npac})$.

Proof: Due to the interesting percolation phenomenon in [3], we know that a node can be either connected to the dominant connected component (denoted as DCC from now on) or individually isolated (not connected to any other nodes) in a Poisson point process of high density. Thus, we have

$$\begin{aligned} P_{com} &\approx 1 - P(\exists \text{ an isolated single active sensor } s \text{ whose sensing disk is not } k\text{-covered by sensors on the DCC}) \\ &> 1 - np \cdot P(s \text{ is isolated and its sensing disk is not } k\text{-covered by sensors on the DCC}) \\ &= 1 - np \cdot P(s \text{ is isolated}) \cdot P_v = 1 - np \cdot e^{-npac} \cdot P_v, \end{aligned} \quad (23)$$

where P_v denotes the probability that s ' sensing disk is not k -covered by sensors on the DCC, given s is isolated. The key step to get P_{com}^L is to find an upper bound of P_v . Using the similar coverage process techniques in the proof of Theorem 1, we can have

$$\begin{aligned} P_v &\leq P(M'_k \geq 2 | s \text{ is isolated}) \leq \frac{1}{2} E(M'_k | s \text{ is isolated}) \\ &= \frac{1}{2} E(\text{number of crossings within } s' \text{ sensing disk} | s \text{ is isolated}) \cdot \\ &P(\text{a crossing within } s' \text{ sensing disk is not } k\text{-covered by sensors on the DCC} | s \text{ is isolated}) \\ &< \frac{1}{2} E(\text{number of crossings within } s' \text{ sensing disk}) \cdot \\ &P(\text{a crossing within } s' \text{ sensing disk is not } k\text{-covered by sensors on the DCC} | s \text{ is isolated}), \end{aligned} \quad (24)$$

where M'_k is the number of crossings in s ' sensing disk (including the disk boundary) that are not k -covered by sensors on the DCC. The expected number of crossings in s ' sensing disk is calculated as

$$\begin{aligned} &E[\text{number of crossings within } s' \text{ sensing disk}] \\ &= E[\text{number of crossings on the boundary of } s' \text{ sensing disk}] + E[\text{number of crossings inside } s' \text{ sensing disk}] \\ &= E[2np \cdot \pi(r_{s,1} + r_{s,2})^2] + E[\frac{1}{2} \cdot (2np \cdot \pi(r_{s,1} + r_{s,2})^2) \cdot np \cdot a_s] \\ &= 8npa'_s + 4n^2p^2a'_sa_s, \end{aligned} \quad (25)$$

where $r_{s,1}$ and $r_{s,2}$ are two sensors' sensing ranges. Next, we will calculate the probability (P_k^{cc}) that, given s is isolated, a crossing within s ' sensing disk its sensing disk is not k -covered by sensors on the DCC. At first, when r_L is larger than \mathcal{L}_U , we can calculate the probability (p_c) that a crossing within s ' sensing disk is covered by a sensor on the DCC, given s is isolated. Because the worse case for the coverage of the crossing q (as shown in the Fig. 1) is that s is just at crossing q and there are no sensors within a distance \mathcal{L}_U to q . Denote the disk area, with

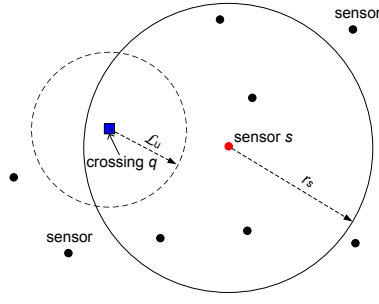


Fig. 1. The red dot represents sensor s and the black dots represent the other sensors. Crossing q , where two sensing disks intersect, is within the sensing disk of s .

a radius \mathcal{L}_U and centered at q , as \mathcal{D}_v . Thus, we may get the lower bound on p_c as below:

$$\begin{aligned}
p_c &= P(\text{sensor } s_0 \text{ is on the DCC}) \times P(\text{sensor } s_0 \text{ covers crossing } q | s \text{ is isolated}) \\
&\geq P(s_0 \text{ is on the DCC}) \times P(s_0 \text{ covers crossing } q | s \text{ is isolated and located at } q) \\
&\geq (P(s_0 \text{ is on the DCC when there is no isolated nodes in network}) + P(s_0 \text{ is on the DCC when there is only one isolated sensor})) \\
&\quad \times \mathbb{E}[|x : (x + S) \cap \{q\} \neq \emptyset \text{ and } x \cap \mathcal{D}_v = \emptyset|] \\
&\geq ((1 - npe^{-npa_c}) + (npe^{-npa_c}(1 - (np - 1)e^{-(np-1)a_c}) \frac{np-1}{np})) \times (a_s - \mathcal{L}_U^2\pi).
\end{aligned} \tag{26}$$

Because, when np is large, the result of $(1 - npe^{-npa_c})$ is very close to that of $(1 - (np - 1)e^{-(np-1)a_c})$, and the result of (e^{-npa_c}) is very small compared with npe^{-npa_c} . Thus, we can have

$$\begin{aligned}
p_c &\geq (a_s - \mathcal{L}_U^2\pi) \times (1 - npe^{-npa_c} + (np - 1)e^{-npa_c}(1 - npe^{-npa_c})) \\
&\approx (a_s - \mathcal{L}_U^2\pi) \times ((1 - npe^{-npa_c}) \cdot (1 + npe^{-npa_c})) \\
&= (a_s - \mathcal{L}_U^2\pi) \times (1 - (np)^2 e^{-2npa_c}).
\end{aligned} \tag{27}$$

Denote $(a_s - \mathcal{L}_U^2\pi) \times (1 - (np)^2 e^{-2npa_c})$ as p'_c . Similar with the Lemma 3, the number of sensors on the DCC that can cover point q , given s is isolated, is Poisson distributed, and, using the above result about p_c and p'_c , we have

$$P_k^{cc} = e^{(-p_c np)} \sum_{j=0}^{k-1} \frac{(p_c np)^j}{j!} \leq e^{-p'_c np} \sum_{j=0}^{k-1} \frac{(p'_c np)^j}{j!}. \tag{28}$$

Using the upper bound on P_k^{cc} , we can write the upper bound on P_v as

$$\begin{aligned}
P_v &\leq \min\{1, \frac{1}{2}(8npa'_s + 4n^2 p^2 a'_s a_s) \cdot e^{-p'_c np} \sum_{j=0}^{k-1} \frac{(p'_c np)^j}{j!}\} \\
&= \min\{1, \frac{1}{2}(8npa'_s + 4n^2 p^2 a'_s a_s) \cdot e^{-\lambda_c} \sum_{j=0}^{k-1} \frac{(\lambda_c)^j}{j!}\},
\end{aligned} \tag{29}$$

where $\lambda_c = np(a_s - \mathcal{L}_U^2\pi)(1 - (np)^2e^{-2npa_c})$. By inserting P_v 's upper bound expression, we can get

$$P_{com} \geq 1 - np \cdot e^{-npa_c} \cdot \min\left\{1, \frac{1}{2}(8npa'_s + 4n^2p^2a'_s a_s) \cdot e^{-\lambda_c} \sum_{j=0}^{k-1} \frac{(\lambda_c)^j}{j!}\right\}, \quad (30)$$

when r_L is no less than \mathcal{L}_U . When r_L is smaller than \mathcal{L}_U , it is very difficult to derive the upper bound on P_v barely based on some simple statistics like mean and variance of r_s and \mathcal{L} . Therefore, we simply choose the upper bound of P_v as 1 and have the lower bound on P_{com} equal to $1 - np \cdot e^{-npa_c}$. \square

REFERENCES

- [1] I. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks," *IEEE Communication Magazine*, vol. 40, no. 8, pp. 102–114, Aug. 2002.
- [2] D. Estrin, R. Govindan, J. Heidemann, and S. Kumar, "Next century challenges: scalable coordination in sensor networks," in *Proc. ACM MobiCom'99*, Seattle, WA, 1999, pp. 263–270.
- [3] R. Meester and R. Roy, *Continuum Percolation*. Cambridge University Press, 1996.
- [4] Z. Kong and E. M. Yeh, "Distributed energy management algorithm for large-scale wireless sensor networks," in *Proc. ACM MobiHoc'07*, Montreal, Quebec, Canada, 2007, pp. 209–218.
- [5] O. Dousse and C. Tavouraris and P. Thiran, "Delay of intrusion detection in wireless sensor networks," in *Proc. ACM MobiHoc'06*, Florence, Italy, 2006, pp. 155–165.
- [6] P. Gupta and P. Kumar, "Critical power for asymptotic connectivity," in *Proc. the 37th IEEE Conference on Decision and Control*, vol. 1, 1998, pp. 1106–1110.
- [7] D. Miorandi and E. Altman, "Coverage and connectivity of ad hoc networks in presence of channel randomness," in *Proc. IEEE INFOCOM'05*, Miami, FL, Mar. 2005, pp. 491–502.
- [8] X. Wang, G. Xing, Y. Zhang, C. Lu, R. Pless, and C. Gill, "Integrated coverage and connectivity configuration in wireless sensor networks," in *Proc. ACM SenSys'03*, Los Angeles, CA, 2003, pp. 28–39.
- [9] B. Liu and D. Towsley, "A study of the coverage of large-scale sensor networks," in *Proc. IEEE MASS'04*, Fort Lauderdale, FL, Oct. 2004, pp. 475–483.
- [10] S. Shakkottai, R. Srikant, and N. Shroff, "Unreliable sensor grids: coverage, connectivity and diameter," in *Proc. IEEE INFOCOM'03*, vol. 2, 2003, pp. 1073–1083.
- [11] S. Kumar, T. Lai, and J. Balogh, "On k-coverage in a mostly sleeping sensor network," in *Proc. ACM MobiCom'04*, Philadelphia, PA, 2004, pp. 144–158.
- [12] F. Ye, G. Zhong, J. Cheng, S. Lu, and L. Zhang, "PEAS: a robust energy conserving protocol for long-lived sensor networks," in *Proc. IEEE ICDCS'03*, Providence, RI, May 2003, pp. 28–37.
- [13] H. Zhang and J. Hou, "On deriving the upper bound of α -lifetime for large sensor networks," in *Proc. ACM MobiHoc'04*, Roppongi Hills, Tokyo, Japan, 2004, pp. 121–132.
- [14] Y. Xu, J. Heidemann, and D. Estrin, "Geography-informed energy conservation for ad hoc routing," in *Proc. ACM MobiCom'01*, Rome, Italy, July 2001.

- [15] C. Hsin and M. Liu, "Network coverage using low duty-cycled sensors: random and coordinated sleep algorithms," in *Proc. IEEE IPSN'04*, Berkeley, CA, Apr. 2004, pp. 433–442.
- [16] C. Gui and P. Mohapatra, "Power conservation and quality of surveillance in target tracking sensor networks," in *Proc. ACM MobiCom'04*, Philadelphia, PA, Sept. 2004.
- [17] P. Hall, *Introduction to the Theory of Coverage Process*. John Wiley and Sons, 1988.
- [18] G. Yang, "Mostly sleeping wireless sensor networks: connectivity, k -coverage, and lifetime," Electrical and Computer Engineering Department, Iowa State University, Tech. Rep., 2007.
- [19] P. Billingsley, *Probability and Measure*. New York: Wiley, 1979.
- [20] T. Rappaport, *Wireless Communications: Principles and Practice*. Prentice Hall, 2001.
- [21] D. Blough and P. Santi, "Investigating upper bounds on network lifetime extension for cell-based energy conservation techniques in stationary ad hoc networks," in *Proc. MobiCom'02*, 2002, pp. 183–192.
- [22] W. Mo, D. Qiao, and Z. Wang, "Lifetime maximization of sensor networks under connectivity and k -coverage constraints," in *Proc. IEEE DCOSS'06*, San Francisco, CA, Jun. 2006, pp. 422–442.