Critical Condition for Connected-k-Coverage in Sensor Networks

Abstract

In this technical report, we provide the omitted calculation steps in our submitted letter.

I. Omitted Calculations in Lemma 2

In our letter, we have omitted the calculation steps for getting P_e's upper bound. Now, we provide it as follow.

Note P_e denotes the conditional probability that, given s is isolated, s' sensing disc is not k-covered by sensors belonging to $G^d_{r_c,n,p}$. We also let N_k denote the number of crossings within s' sensing disc (including the disc boundary) that are not k-covered by sensors on $G_{(r_c,n,p)}$. By applying Markov's inequality and the crossing technique in [1], we can have

$$\begin{aligned}
&P_e \leqslant P(N_k \geqslant 2 | s \text{ is isolated}) \leqslant \frac{\mathrm{E}(N_k | s \text{ is isolated})}{2} \\
&\quad = \frac{1}{2}\mathrm{E}(\text{number of crossings within } s\text{' sensing disc} | s \text{ is isolated}) \cdot \\
&\quad P(\text{a crossing within } s\text{' sensing disc is not } k\text{-covered by sensors on the } G_{(r_c,n,p)} | s \text{ is isolated}) \\
&\quad < \frac{1}{2}\mathrm{E}(\text{number of crossings within } s\text{' sensing disc}) \cdot \\
&\quad P(\text{a crossing within } s\text{' sensing disc is not } k\text{-covered by sensors on the } G_{(r_c,n,p)} | s \text{ is isolated}),
\end{aligned} \tag{1}$$

The expected number of crossings in s' sensing disc is calculated as follows:

$$\begin{aligned}
&\mathrm{E}[\text{number of crossings within } s\text{' sensing disc}] \\
=&\mathrm{E}[\text{number of crossings on the boundary of } s\text{' sensing disc}] + \mathrm{E}[\text{number of crossings inside } s\text{' sensing disc}] \\
=&2np\pi(2r_s)^2 + \pi r_s^2 \times (2r_s)^2 \pi np \times np \\
=&8np\pi r_s^2 + 4(np\pi r_s^2)^2.
\end{aligned} \tag{2}$$

Next, we will calculate the probability (P^{cc}_k) that, given s is isolated, a crossing within s' sensing disc is not k-covered by sensors belonging to $G_{(r_c,n,p)}$. We have already denoted the probability that a sensor belongs to $G_{(r_c,n,p)}$ as P_a before. Also, the probability that a sensor belongs to $G_{(r_c,n,p)}$ and can cover a crossing within s' sensing disc, given that s has no communication neighbors within r_c, can be lower bounded by $\pi(r_s^2 - r_c^2)P_a$. In addition, the number of such sensors that belong to $G_{(r_c,n,p)}$ and cover the crossing follows a Poisson distribution. Therefore, P_k^{cc} can be upper bounded by $e^{-np\pi(r_s^2-r_c^2)P_a} \sum_{i=0}^{k-1} \frac{(np\pi(r_s^2-r_c^2)P_a)^i}{i!}$.

Finally, by plugging the above results into (1), we can upper bound P_e by $(4np\pi r_s^2 + 2(np\pi r_s^2)^2)e^{-np\pi(r_s^2-r_c^2)P_a} \sum_{i=0}^{k-1} \frac{(np\pi(r_s^2-r_c^2)P_a)^i}{i!}$.

REFERENCES

[1] P. Hall, *Introduction to the Theory of Coverage Process.* John Wiley and Sons, 1988.