Assigned: 1-20-16

Recall that for $\vec{x}, \vec{y} \in \mathbb{R}^3$, $|\vec{x} \cdot \vec{y}| \leq \|\vec{x}\|\|\vec{y}\|$. This is called the Cauchy-Schwarz inequality, and is valid for $\vec{x}, \vec{y} \in \mathbb{R}^d$ for any dimension $d$. Use this to prove that for any $m \times n$ matrix $M$, there exists a constant $K$ such that for all $\vec{x} \in \mathbb{R}^n$, 

$$\|M\vec{x}\| \leq K\|\vec{x}\|.$$ 

In fact, $K$ can be taken to be the following:

$$K = \sqrt{\sum_{j=1}^m \sum_{k=1}^n |M_{jk}|^2}.$$ 

The notation here means

$$M = \begin{pmatrix} M_{11} & M_{12} & \ldots & M_{1n} \\ M_{21} & M_{22} & \ldots & M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{m1} & M_{m2} & \ldots & M_{mn} \end{pmatrix}.$$