Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}^3$, and write $f(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{pmatrix}$. Fix $t_0 \in \mathbb{R}$; prove that $f$ is differentiable at $t_0$ if and only if $f_1$, $f_2$, and $f_3$ are differentiable at $t_0$. If $f$ is differentiable at $t_0$, calculate the linear transformation $T$. 
