Exercise
Assigned: Jan. 20
Due: Jan. 23

Here are a few properties that we've been told are true, and here are the proofs:

1. $-x = (-1)x$

   Proof. By definition of additive inverses, $x + (-x) = 0$. However, we also have that $x+(-1)x = x(1+(-1))$ by the distributivity property, but then $x+(-1)x = x\cdot 0 = 0$ by the statement proven in class. Since both are equal to 0, we obtain $x+(-x) = x+(-1)x$, so adding the inverse of x to both sides we conclude that $(-x) = (-1)x$. $\square$

2. If $x > 0$ and $y < 0$, then $xy < 0$.

   Proof. By the trichotomy property, we have that $-y > 0$, and therefore by the closure under multiplication property, $x\cdot(-y) > 0$. By the above equality, $x\cdot(-y) = x\cdot(-1)\cdot y = (-1)x \cdot y$ by the associativity and commutativity properties, and thus $x \cdot (-y) = -xy$. Since $-xy > 0$, again by the trichotomy property, we obtain that $xy < 0$. $\square$

Use these facts to prove parts b), c), d), e) of Proposition 1.46 on page 17.