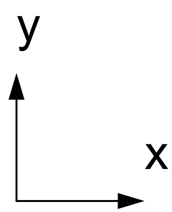
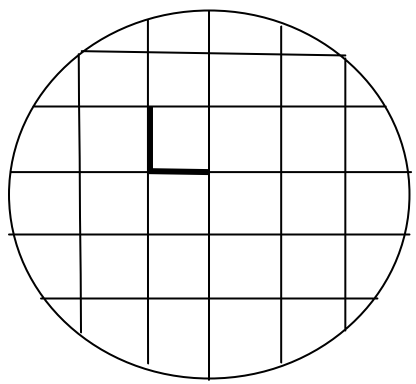
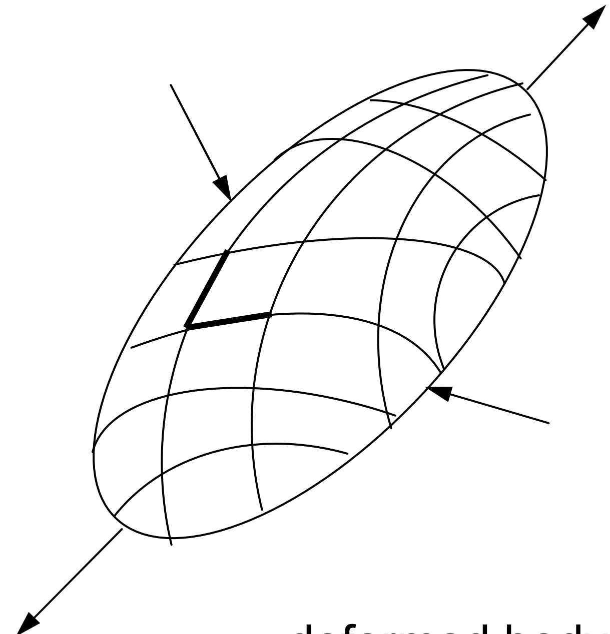
Strains
(2-D deformation -plane strain)

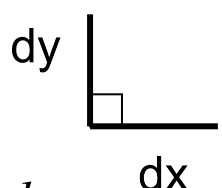


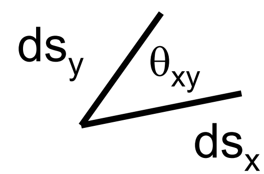




deformed body

undeformed body





normal strains $e_{xx} = \frac{ds_x - dx}{dx}$

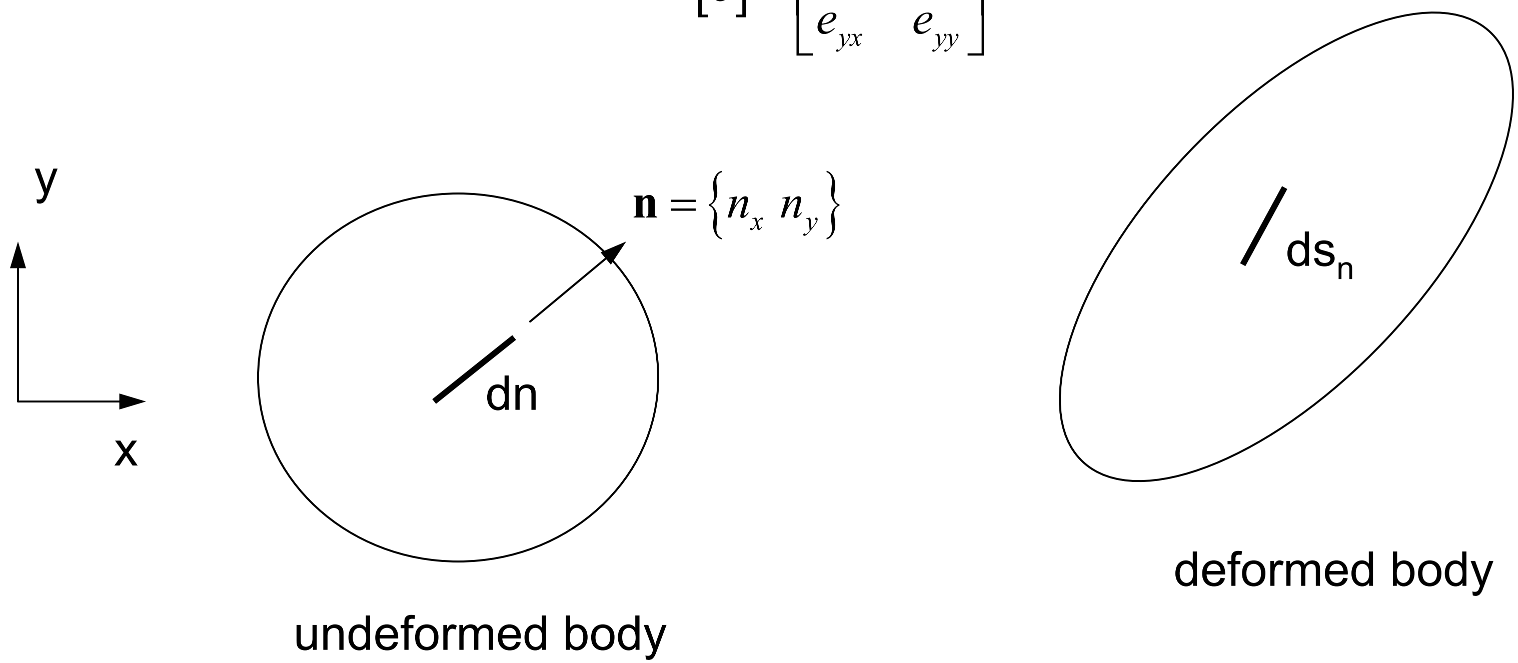
$$e_{yy} = \frac{ds_y - dy}{dy}$$

engineering shear strain $\gamma_{xy} = \frac{\pi}{2} - \theta_{xy}$

tensor shear strain $e_{xy} = \frac{\gamma_{xy}}{2}$

state of strain (plane strain)

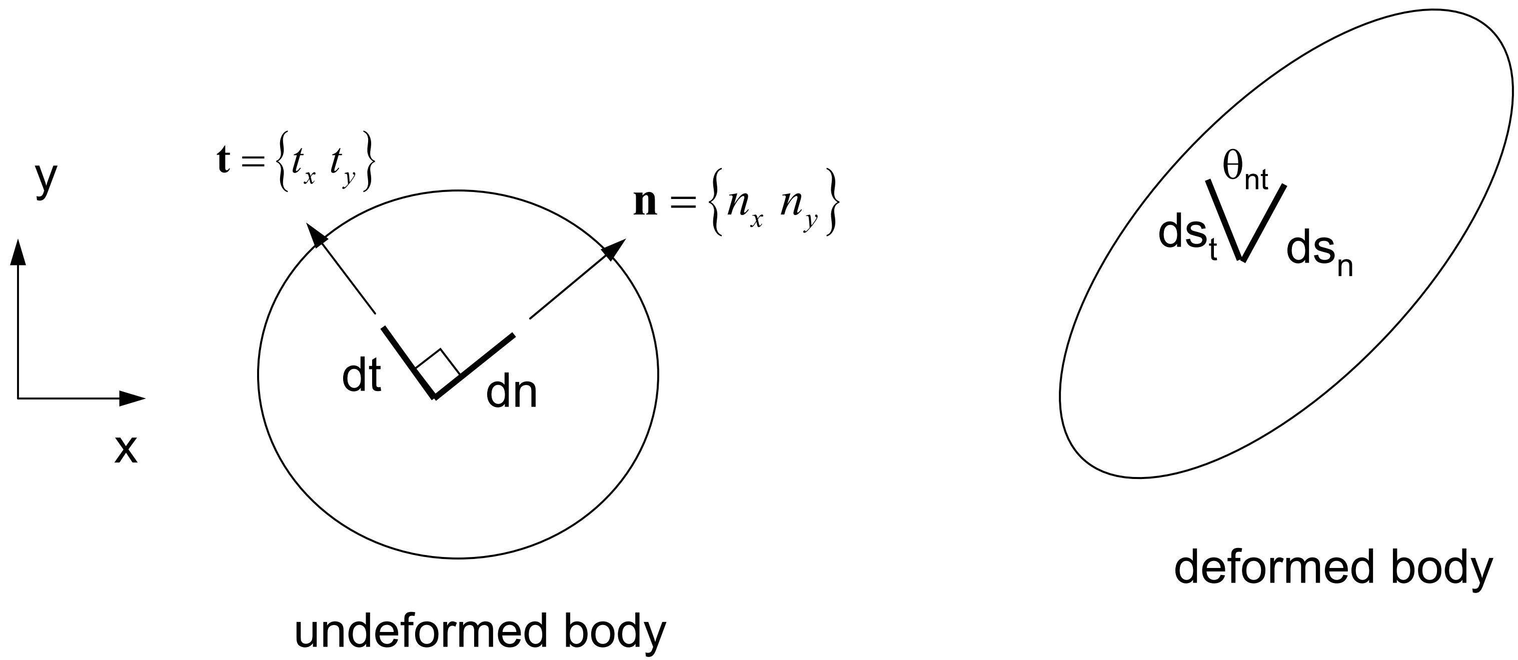
$$[e] = \begin{bmatrix} e_{xx} & e_{xy} \\ e_{yx} & e_{yy} \end{bmatrix}$$



$$\mathbf{n} = \{ n_x \ n_y \}$$

$$e_{nn} = \frac{ds_n - dn}{dn}$$

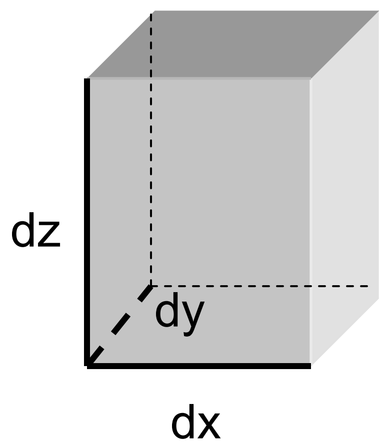
$$e_{nn} = e_{xx} n_x^2 + e_{yy} n_y^2 + 2 e_{xy} n_x n_y$$



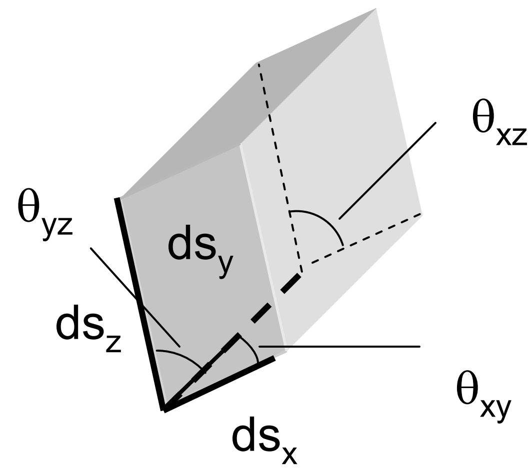
$$e_{nt} = \frac{\gamma_{nt}}{2} = \frac{1}{2}\left(\frac{\pi}{2} - \theta_{nt}\right)$$

$$\begin{aligned} e_{nt} &= e_{xx} n_x t_x + e_{yy} n_y t_y \\ &\quad + e_{xy}\left(n_x t_y + n_y t_x\right) \end{aligned}$$

3-D strains



undeformed body



deformed body

state of strain

$$\begin{bmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{bmatrix}$$

3-D Strain transformation relations

$$e_{nn} = e_{xx}n_x^2 + e_{yy}n_y^2 + e_{zz}n_z^2$$
$$+2e_{xy}n_xn_y$$
$$+2e_{xz}n_xn_z$$
$$+2e_{yz}n_yn_z$$

$$e_{nt} = e_{xx}n_xt_x + e_{yy}n_yt_y + e_{zz}n_zt_z$$
$$+e_{xy}\left(n_xt_y + n_yt_x\right)$$
$$+e_{xz}\left(n_xt_z + n_zt_x\right)$$
$$+e_{yz}\left(n_yt_z + n_zt_y\right)$$

or

$$e_{nn} = \{n\}[e]\{n\}^T \qquad e_{nt} = \{n\}[e]\{t\}^T = \{t\}[e]\{n\}^T$$

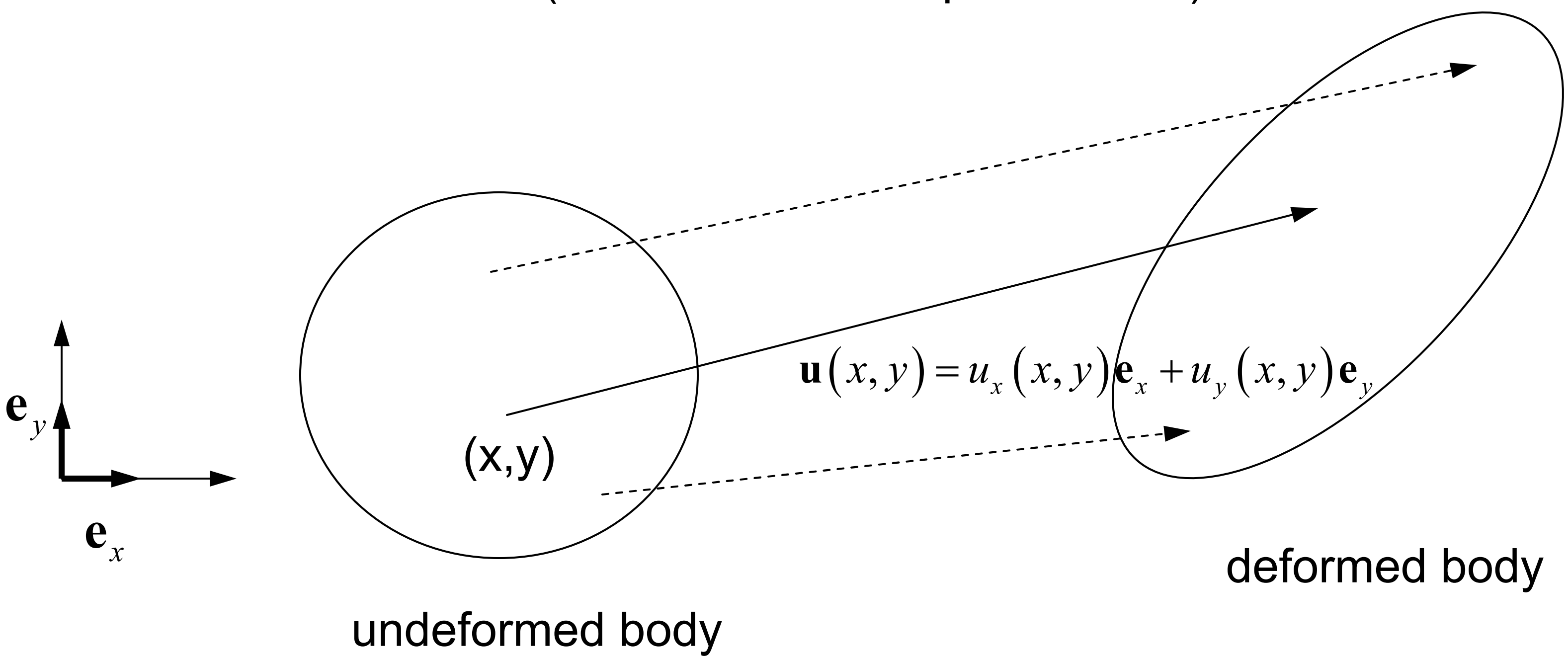
$$\begin{bmatrix} e_{nn} & e_{nt} & e_{nv} \\ e_{tn} & e_{tt} & e_{tv} \\ e_{vn} & e_{vt} & e_{vv} \end{bmatrix} = \begin{bmatrix} n_x & n_y & n_z \\ t_x & t_y & t_z \\ v_x & v_y & v_z \end{bmatrix} \begin{bmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{y} & e_{zz} \end{bmatrix} \begin{bmatrix} n_x & t_x & v_x \\ n_y & t_y & v_y \\ n_z & t_z & v_z \end{bmatrix}$$

or

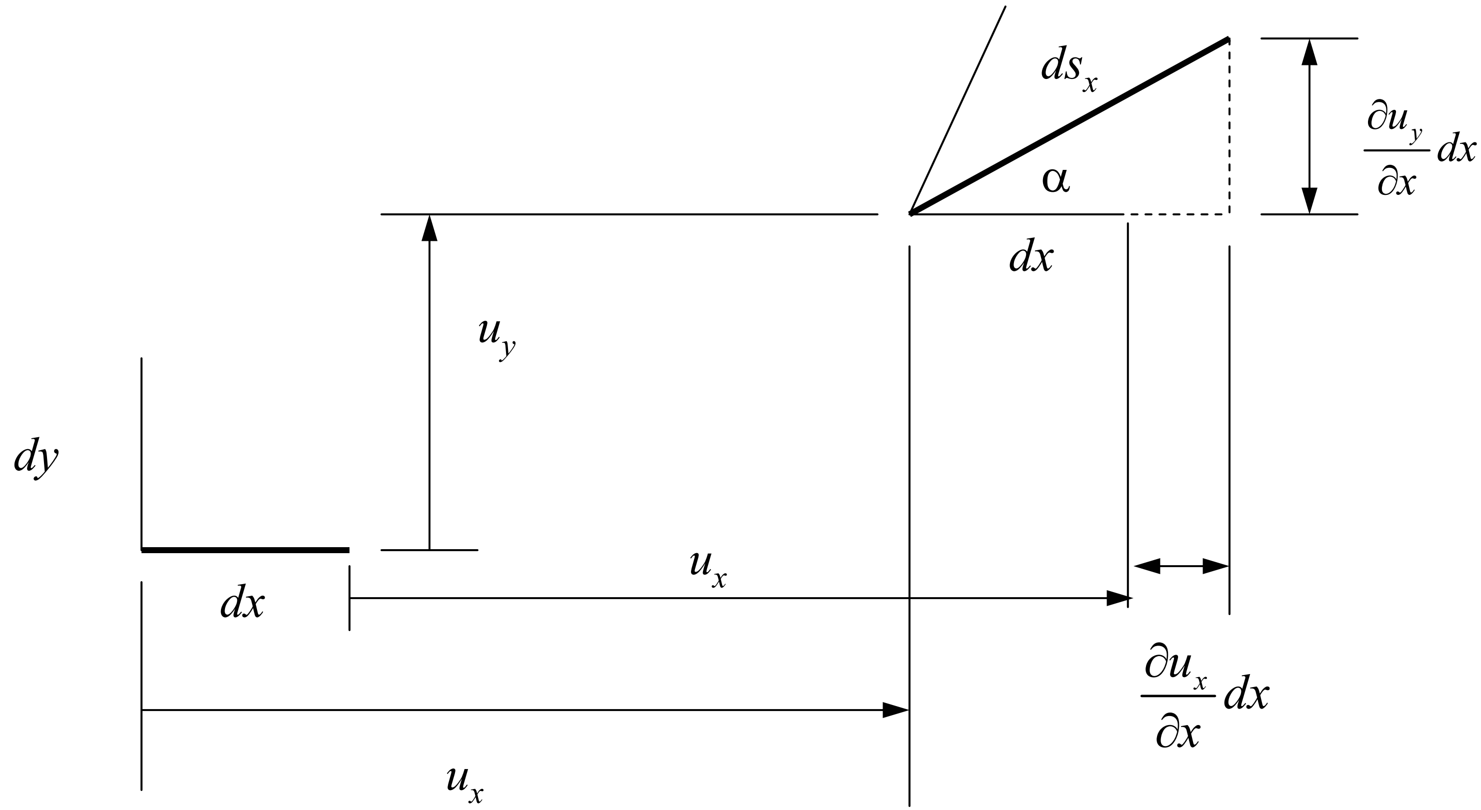
$$[e'] = [l]^T [e][l]$$

Strains and displacement gradients

(2-D deformation - plane strain)



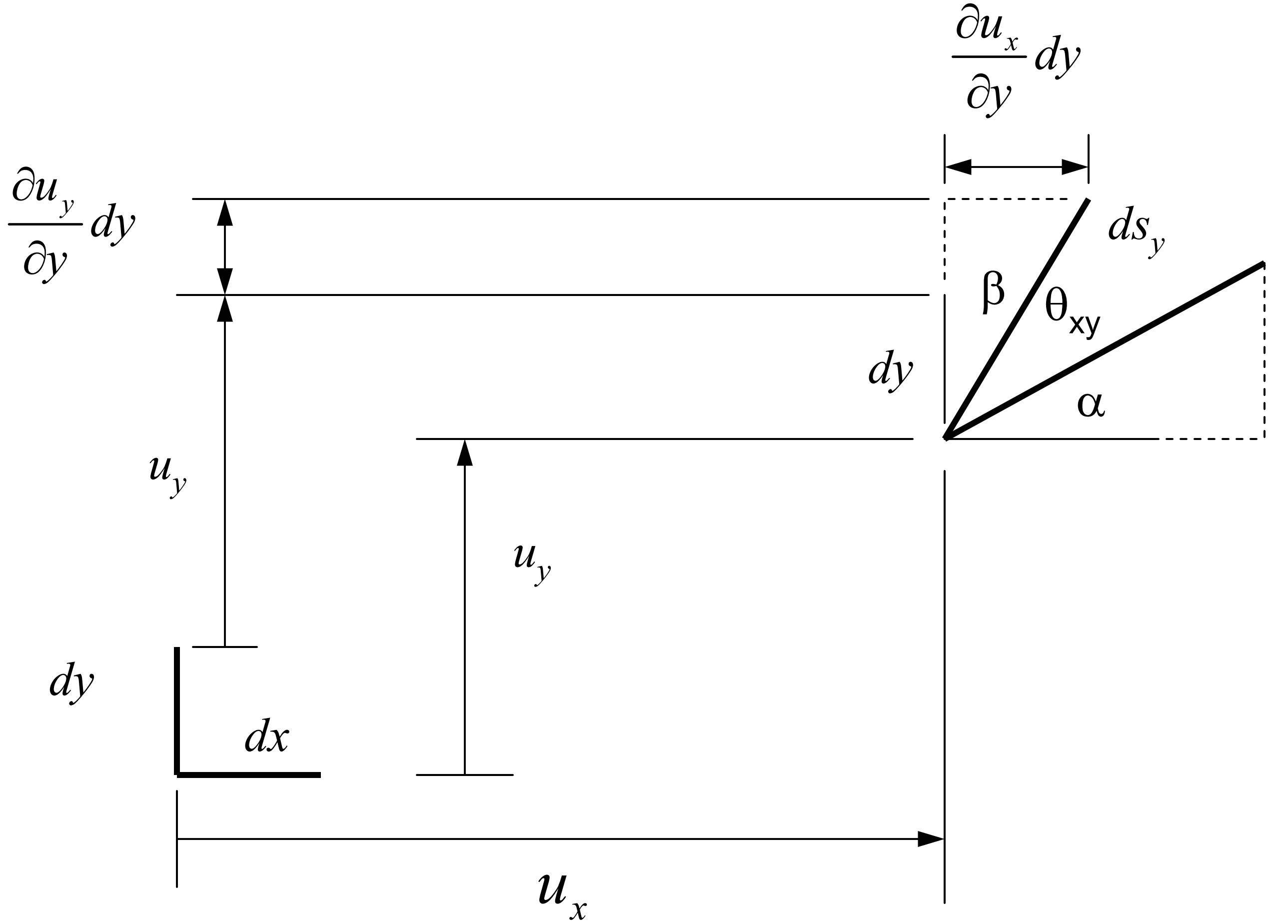
u(x,y) …. displacement vector



$$e_{xx} = \frac{ds_x - dx}{dx} = \frac{\sqrt{\left(dx + \frac{\partial u_x}{\partial x}dx\right)^2 + \left(\frac{\partial u_y}{\partial x}\right)^2} - dx}{dx}$$

$$\alpha \cong \tan\alpha = \frac{\frac{\partial u_y}{\partial x}dx}{dx + \frac{\partial u_x}{\partial x}dx} \cong \frac{\partial u_y}{\partial x}$$

$$\cong \frac{\sqrt{dx^2 + 2dx\frac{\partial u_x}{\partial x} + H.O.T} - dx}{dx} \cong \frac{dx\sqrt{1 + 2\frac{\partial u_x}{\partial x}} - dx}{dx} = \frac{\partial u_x}{\partial x}$$



$$e_{yy} = \frac{ds_y - dy}{dy} \cong \frac{dy\sqrt{1 + 2\frac{\partial u_y}{\partial y}} - dy}{dy} = \frac{\partial u_y}{\partial y}$$

$$\beta \cong \tan\beta \cong \frac{\partial u_x}{\partial y}$$

$$e_{xy} = \frac{1}{2}\left(\frac{\pi}{2} - \theta_{xy}\right) = \frac{1}{2}(\alpha + \beta)$$

$$= \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

2-D deformation strain – displacement relations

$$e_{xx} = \frac{\partial u_x}{\partial x}$$

$$e_{yy} = \frac{\partial u_y}{\partial y}$$

$$e_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

3-D deformation strain –displacement relations

$$e_{xx} = \frac{\partial u_x}{\partial x}, \quad e_{yy} = \frac{\partial u_y}{\partial y}, \quad e_{zz} = \frac{\partial u_z}{\partial z}$$

$$e_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

$$e_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)$$

$$e_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)$$

Remarks on Finite Strains

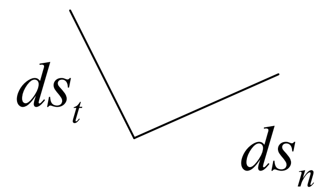
Normal Strain

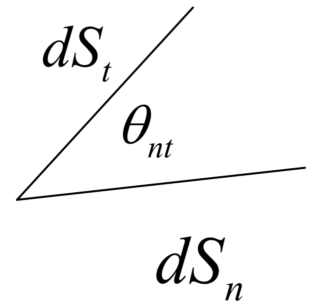
$$E_{nn} = \frac{dS_n^2 - ds_n^2}{2ds_n^2}$$

$$= \frac{dS_n - ds_n}{ds_n} \frac{dS_n + ds_n}{2ds_n}$$

$$\cong \frac{dS_n - ds_n}{ds_n} = e_{nn}$$

for small strains





Shear Strain

$$E_{nt} = \frac{1}{2}\sqrt{1+2E_{nn}}\sqrt{1+2E_{tt}}\,\sin\left(\frac{\pi}{2} - \theta_{nt}\right)$$

$$\cong \frac{1}{2}\left(\frac{\pi}{2} - \theta_{nt}\right) = e_{nt} = \frac{\gamma_{nt}}{2}$$

for small strains

Transformations

$$E_{nt} = E_{xx} n_x t_x + E_{yy} n_y t_y + E_{zz} n_z t_z$$
$$+E_{xy}\left(n_x t_y + n_y t_x\right)$$
$$+E_{xz}\left(n_x t_z + n_z t_x\right)$$
$$+E_{zy}\left(n_z t_y + n_y t_z\right)$$

Strain-Displacement relations

$$E_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} + \underbrace{\frac{\partial u_x}{\partial x}\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial x}\frac{\partial u_z}{\partial y}}_{\text{non-linear}}\right)$$

etc.

non-linear