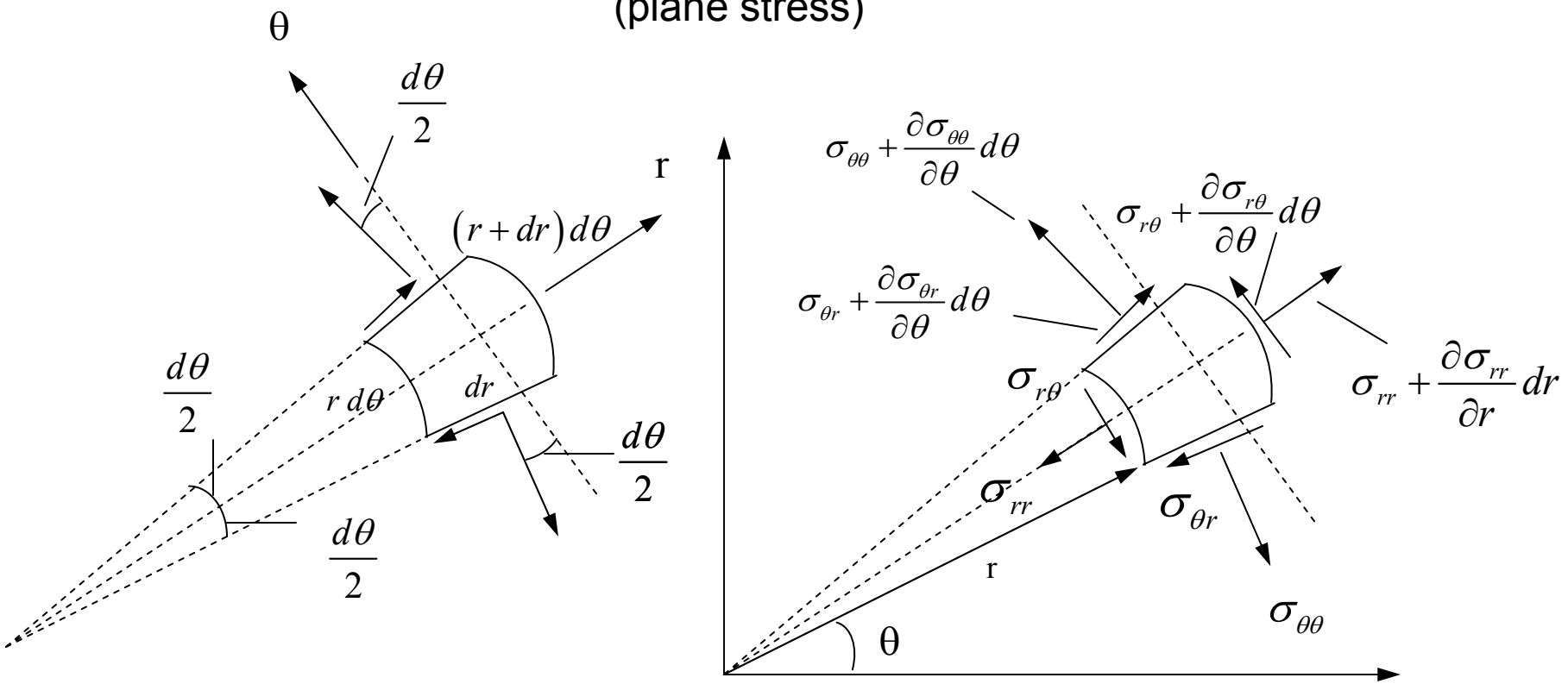
Equilibrium equation in polar coordinates (plane stress)



$$\sum F_r = 0$$

$$\left(\sigma_{rr}+\frac{\partial\sigma_{rr}}{\partial r}dr\right)(r+dr)\,d\theta dz-\sigma_{rr}rd\theta dz-\sigma_{\theta r}drdz\cos\left(\frac{d\theta}{2}\right)+\left(\sigma_{\theta r}+\frac{\partial\sigma_{\theta r}}{\partial\theta}d\theta\right)drdz\cos\left(\frac{d\theta}{2}\right)$$

$$-\sigma_{\theta\theta}drdz\sin\left(\frac{d\theta}{2}\right)-\left(\sigma_{\theta\theta}+\frac{\partial\sigma_{\theta\theta}}{\partial\theta}d\theta\right)drdz\sin\left(\frac{d\theta}{2}\right)=0$$

$$\simeq 1 \qquad\qquad \simeq 1$$

$$\left(\sigma_{rr}+\frac{\partial\sigma_{rr}}{\partial r}dr\right)(r+dr)\,d\theta dz-\sigma_{rr}rd\theta dz-\sigma_{\theta r}drdz\cos\left(\frac{d\theta}{2}\right)+\left(\sigma_{\theta r}+\frac{\partial\sigma_{\theta r}}{\partial\theta}d\theta\right)drdz\cos\left(\frac{d\theta}{2}\right)$$

$$-\sigma_{\theta\theta}drdz\sin\left(\frac{d\theta}{2}\right)-\left(\sigma_{\theta\theta}+\frac{\partial\sigma_{\theta\theta}}{\partial\theta}d\theta\right)drdz\sin\left(\frac{d\theta}{2}\right)=0$$

$$\cong\frac{d\theta}{2} \qquad\qquad \cong\frac{d\theta}{2}$$

H.O.T

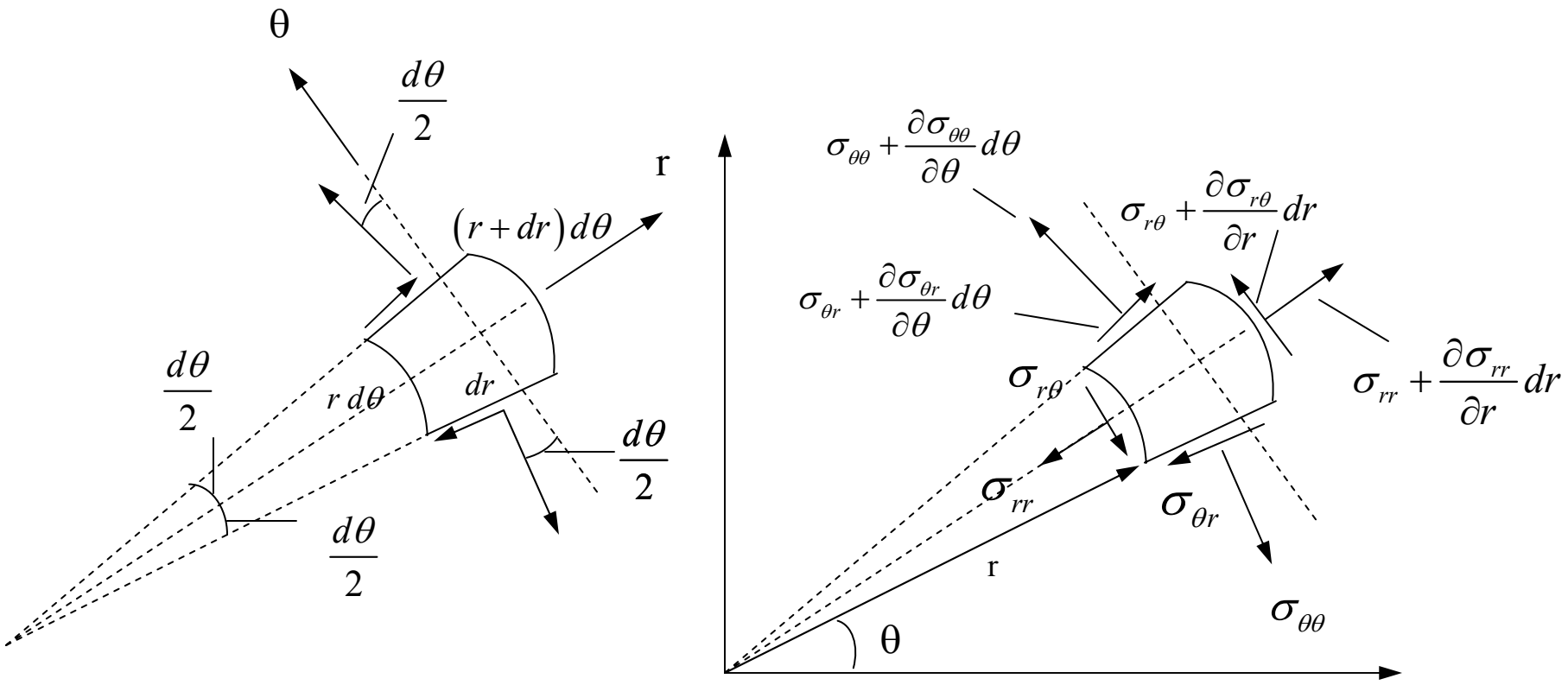
$$\sigma_{rr}rd\theta dz+\sigma_{rr}drd\theta dz+\frac{\partial\sigma_{rr}}{\partial r}rdrd\theta dz+\frac{\partial\sigma_{rr}}{\partial r}(dr)^2\,d\theta dz-\sigma_{rr}rd\theta dz$$

$$-\sigma_{\theta r}drdz+\sigma_{\theta r}drdz+\frac{\partial\sigma_{\theta r}}{\partial\theta}drd\theta dz-\sigma_{\theta\theta}drdz\frac{d\theta}{2}-\sigma_{\theta\theta}drdz\frac{d\theta}{2}-\frac{\partial\sigma_{\theta\theta}}{\partial\theta}drdz\frac{(d\theta)^2}{2}=0$$

H.O.T

$$\left(\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta r}}{\partial\theta}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}\right)rdrd\theta dz=0$$

$$\Rightarrow \qquad \frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta r}}{\partial\theta}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=0$$



$$\sum F_\theta = 0$$

$$\left(\sigma_{\theta\theta}+\frac{\partial\sigma_{\theta\theta}}{\partial\theta}d\theta\right)drdz\cos\left(\frac{d\theta}{2}\right)-\sigma_{\theta\theta}drdz\cos\left(\frac{d\theta}{2}\right)+\left(\sigma_{r\theta}+\frac{\partial\sigma_{r\theta}}{\partial r}dr\right)(r+dr)\,d\theta dz-\sigma_{r\theta}rd\theta dz$$

$$+\left(\sigma_{\theta r}+\frac{\partial\sigma_{\theta r}}{\partial\theta}d\theta\right)drdz\sin\left(\frac{d\theta}{2}\right)+\sigma_{\theta r}drdz\sin\left(\frac{d\theta}{2}\right)=0$$

$$\simeq 1 \qquad\qquad \simeq 1$$

$$\left(\sigma_{\theta\theta}+\frac{\partial\sigma_{\theta\theta}}{\partial\theta}d\theta\right)drdz\cos\left(\frac{d\theta}{2}\right)-\sigma_{\theta\theta}drdz\cos\left(\frac{d\theta}{2}\right)+\left(\sigma_{r\theta}+\frac{\partial\sigma_{r\theta}}{\partial r}dr\right)(r+dr)\,d\theta dz-\sigma_{r\theta}rd\theta dz$$

$$+\left(\sigma_{\theta r}+\frac{\partial\sigma_{\theta r}}{\partial\theta}d\theta\right)drdz\sin\left(\frac{d\theta}{2}\right)+\sigma_{\theta r}drdz\sin\left(\frac{d\theta}{2}\right)=0$$

$$\cong\frac{d\theta}{2}$$

$$\sigma_{\theta\theta}drdz+\frac{\partial\sigma_{\theta\theta}}{\partial\theta}drd\theta dz-\sigma_{\theta\theta}drdz+\sigma_{r\theta}rd\theta dz+\sigma_{r\theta}drd\theta dz+\frac{\partial\sigma_{r\theta}}{\partial r}rdrd\theta dz-\sigma_{r\theta}rd\theta dz$$

$$+\sigma_{\theta r}drdz\left(\frac{d\theta}{2}\right)+\frac{\partial\sigma_{\theta r}}{\partial\theta}drdz\frac{(d\theta)^2}{2}+\sigma_{\theta r}drdz\left(\frac{d\theta}{2}\right)=0$$

H.O.T

$$\left(\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{2\sigma_{r\theta}}{r}\right)rdrd\theta dz=0$$

$$\Rightarrow\qquad\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{2\sigma_{r\theta}}{r}=0$$

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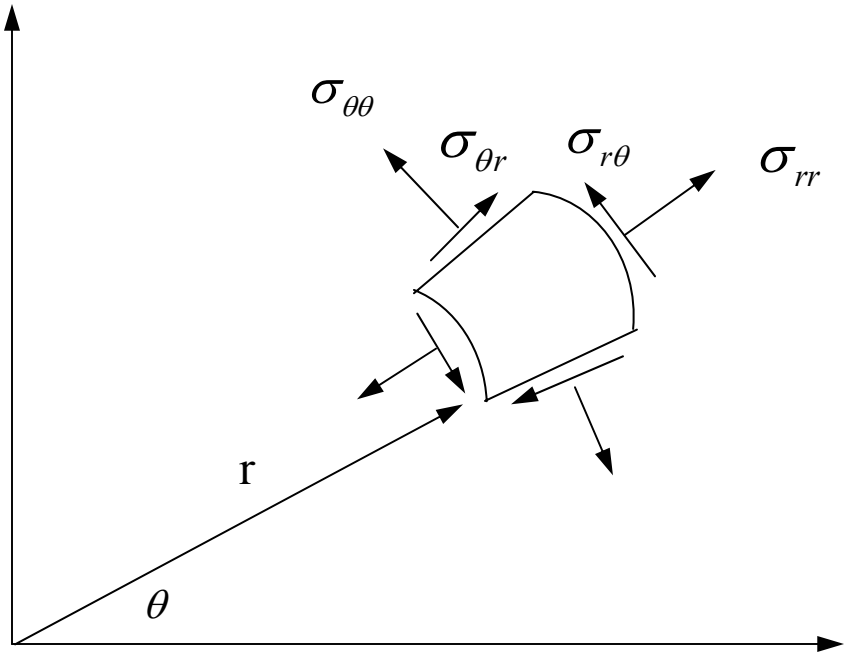

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta r}}{\partial \theta} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{2\sigma_{r\theta}}{r} = 0$$