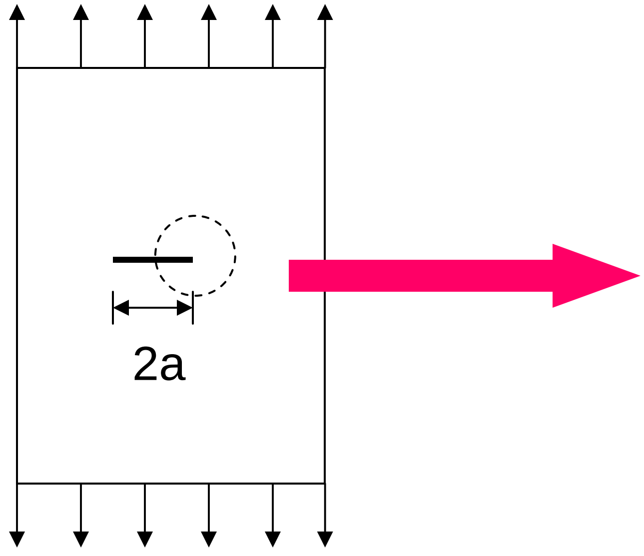
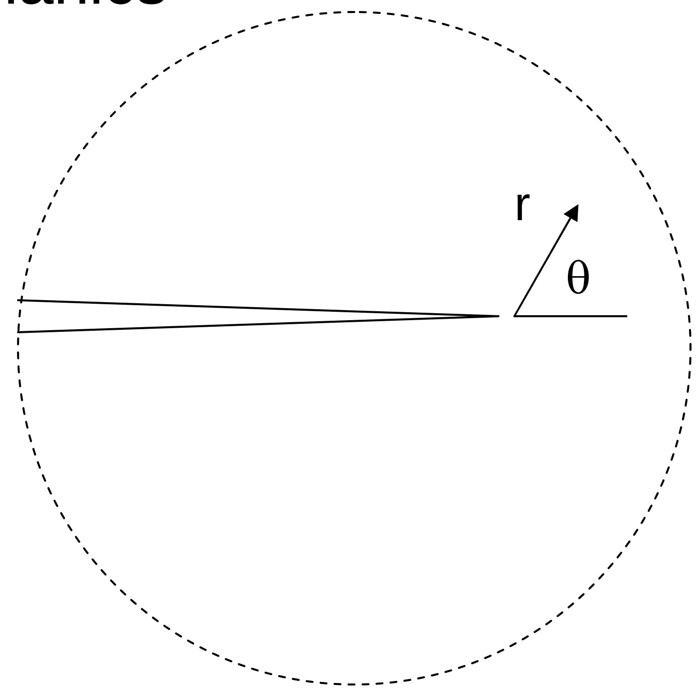
Fracture Mechanics



through-thickness crack



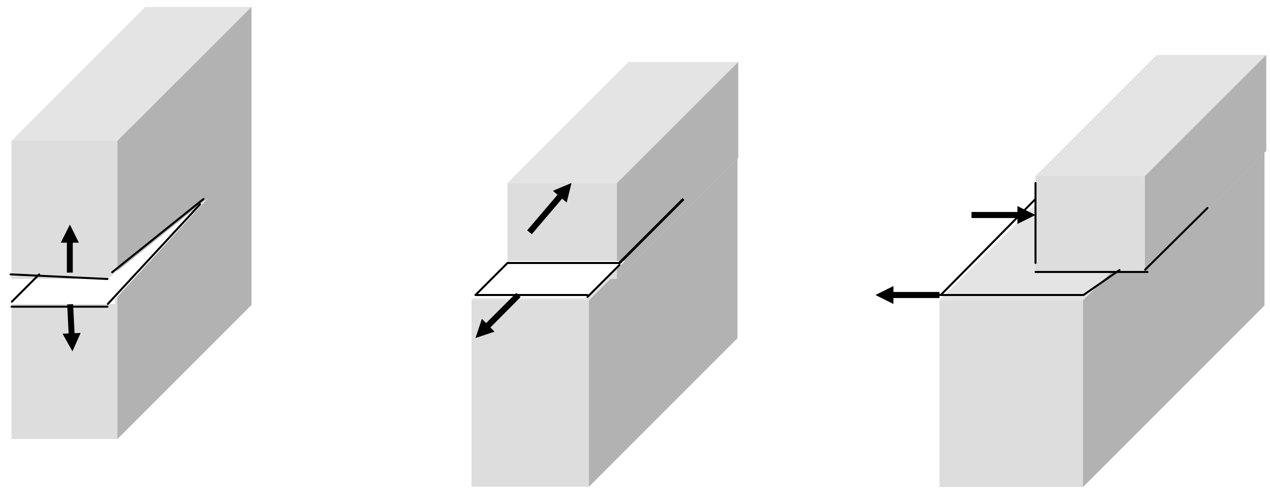
near the crack tip, if behavior is elastic, stresses (for r small) look like

$$\sigma \cong \frac{K_I g(\theta)}{\sqrt{2\pi r}}$$

K_I is the stress intensity factor for the crack in an opening mode

The dimension of the stress intensity is $stress\sqrt{length}$

There are three different crack modes (types of crack deformation)
Each mode has its own stress intensity factor



mode I — K_I — opening mode

mode II — K_{II} — shearing mode

mode III — K_{III} — tearing mode

Most cracks tend to propagate in an opening mode so the majority of fracture analyses assume that mode

Stress intensity factors are written in the form

$$K = f\left(g, a\right)\sigma\sqrt{\pi a}$$

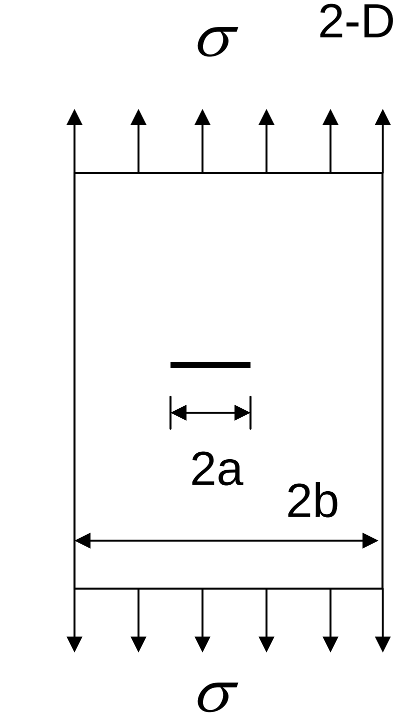
σ is a reference stress value

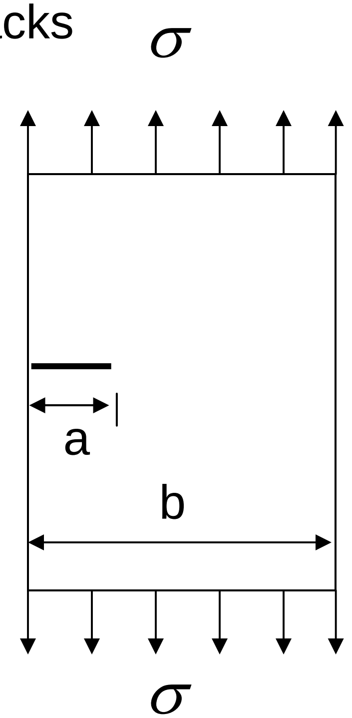
a is the crack half-length (radius for a 3-D penny-shaped crack)

$f(g,a)$ is a non-dimensional ***configuration factor*** that depends on the geometry of a component the crack is in and the crack length

Stress intensity configuration factors have been tabulated for many different geometries.

2-D (through thickness) cracks





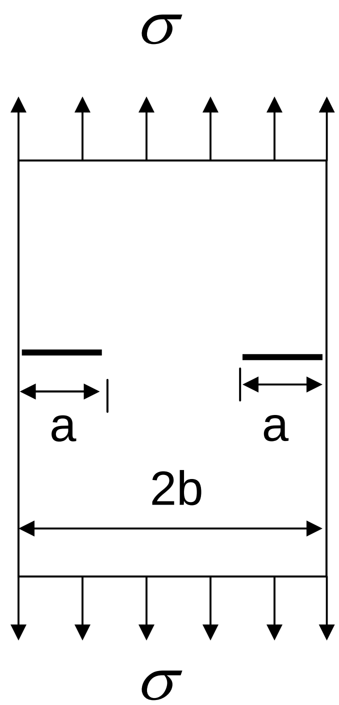
$$K_I = f\left(g,a\right)\sigma\sqrt{\pi a}$$

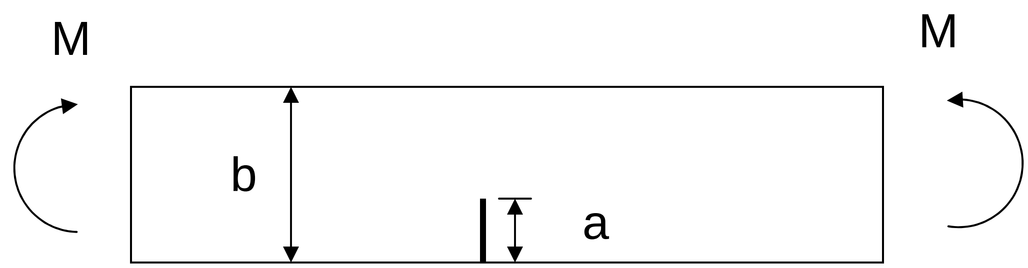
$$f\left(g,a\right) = \sqrt{\sec\left(\frac{\pi a}{2b}\right)}$$

$$\cong 1\left(b >> a\right)$$

$$K_I = f\left(g,a\right)\sigma\sqrt{\pi a}$$

$$f\left(g,a\right) = 1.12 - 0.231\left(a/b\right) + 10.55\left(a/b\right)^2$$
$$-21.72\left(a/b\right)^3 + 30.39\left(a/b\right)^4$$





t = thickness
$\sigma = 6M / tb^2$ = max tensile bending stress

$$K_I = f(g,a)\sigma\sqrt{\pi a}$$

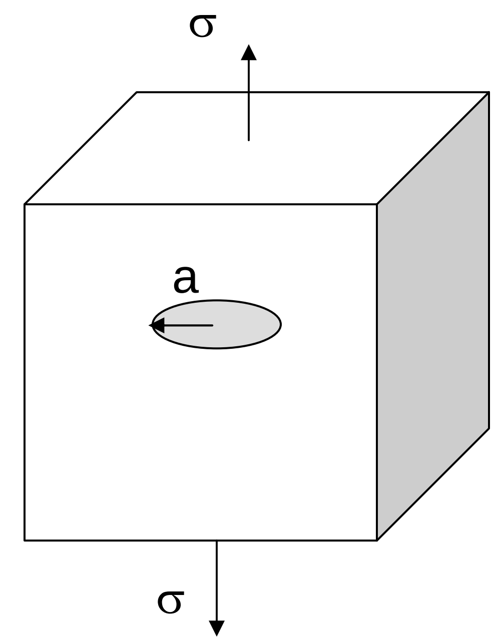
$$f(g,a) = 1.12 + 0.203(a/b) - 1.197(a/b)^2 + 1.930(a/b)^3$$

$$K_I = f(g,a)\sigma\sqrt{\pi a}$$

$$f(g,a) = 1.122 - 1.40(a/b) + 7.33(a/b)^2 - 13.08(a/b)^3 + 14.0(a/b)^4$$

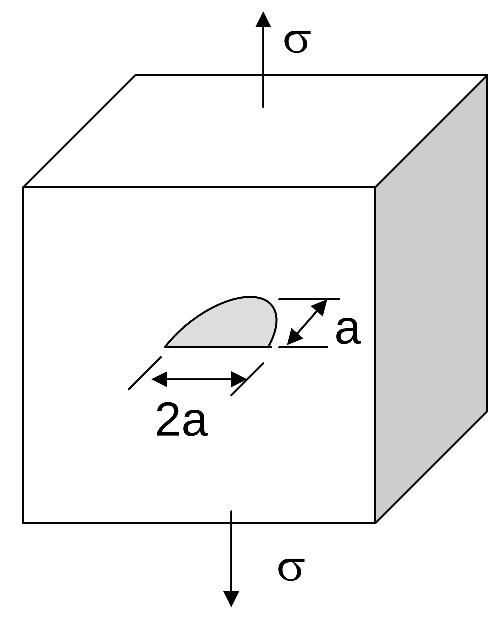
3-D cracks

embedded penny-shaped crack in tension



$$K_I = \frac{2\sigma}{\pi}\sqrt{\pi a}$$

surface-breaking semi-circular (thumbnail) crack in tension



$$K_I = 1.12\left(\frac{2\sigma}{\pi}\sqrt{\pi a}\right)$$

The reason that the stress intensity factor is useful is because fracture (rapid crack growth to failure) occurs when K exceeds a certain critical value, K_c, called the ***fracture toughness***

In general, $K_c = K_c$(specimen thickness, temperature, environmental conditions, etc.)

As the specimen thickness increases, K_c decreases until it reaches a constant value called the ***plane strain fracture toughness***.

For a crack in opening mode I, the plane strain fracture toughness is usually written as K_{IC}

The size of the crack at the plane strain toughness is called the critical flaw size, a_c, where

$$K_{IC} = f\left(g, a_c\right)\sigma\sqrt{\pi a_c}$$

To obtain a rough estimate of the critical flaw size:

1. assume a small half-penny shaped crack on the surface of the component. If the crack is stressed in an opening mode then

$$K_I = 1.12\left(\frac{2\sigma}{\pi}\right)\sqrt{\pi a}$$

2. fracture occurs when $K_I = K_{IC}$, $a = a_C$
so solving for a_C

$$a_c = \frac{1}{\pi}\left[\frac{K_{IC}}{0.713\sigma}\right]^2$$

3. Now suppose that the structure is loaded so that $\sigma = \sigma_{yield}$

Then

$$a_c = \frac{1}{1.6}\left(\frac{K_{IC}}{\sigma_{yield}}\right)^2$$

Example 1: 4340 steel (an airframe material)

$$\sigma_Y = 240\ ksi$$

$$K_{IC} = 50\ ksi\sqrt{in}$$

$$\Rightarrow \quad a_c = 0.027\ in.$$

Example 2: A533B steel (a nuclear pressure vessel material)

$$\sigma_Y = 60\ ksi$$

$$K_{IC} = 140\ ksi\sqrt{in}$$

$$\Rightarrow \quad a_c = 3.40\ in.$$

Thus, the critical crack size can vary significantly with the choice of material

To guarantee the safety of critical components, such as pressure vessels, aircraft engines and airframes, nuclear reactors, etc. those components are inspected periodically with nondestructive evaluation (NDE) techniques to find (and size) any crack that might exist.

See the sections on NDE for a brief introduction to that area

Another use of fracture mechanics is to determine the rate of growth of a crack from a benign size to a critical size and to estimate the remaining useful safe life of a component.

For example, for fatigue crack growth under a cyclic load, if the stress changes from $\sigma_{\min}$ to $\sigma_{\max}$ the stress intensity factor will change from $K_{\min}$ to $K_{\max}$ since

$$K_{\max,\min} = f\left(g,a\right)\sigma_{\max,\min}\sqrt{\pi a}$$

A commonly used crack growth law is the ***Paris Law***:

$$\frac{da}{dN} = C\Delta K^{m}$$

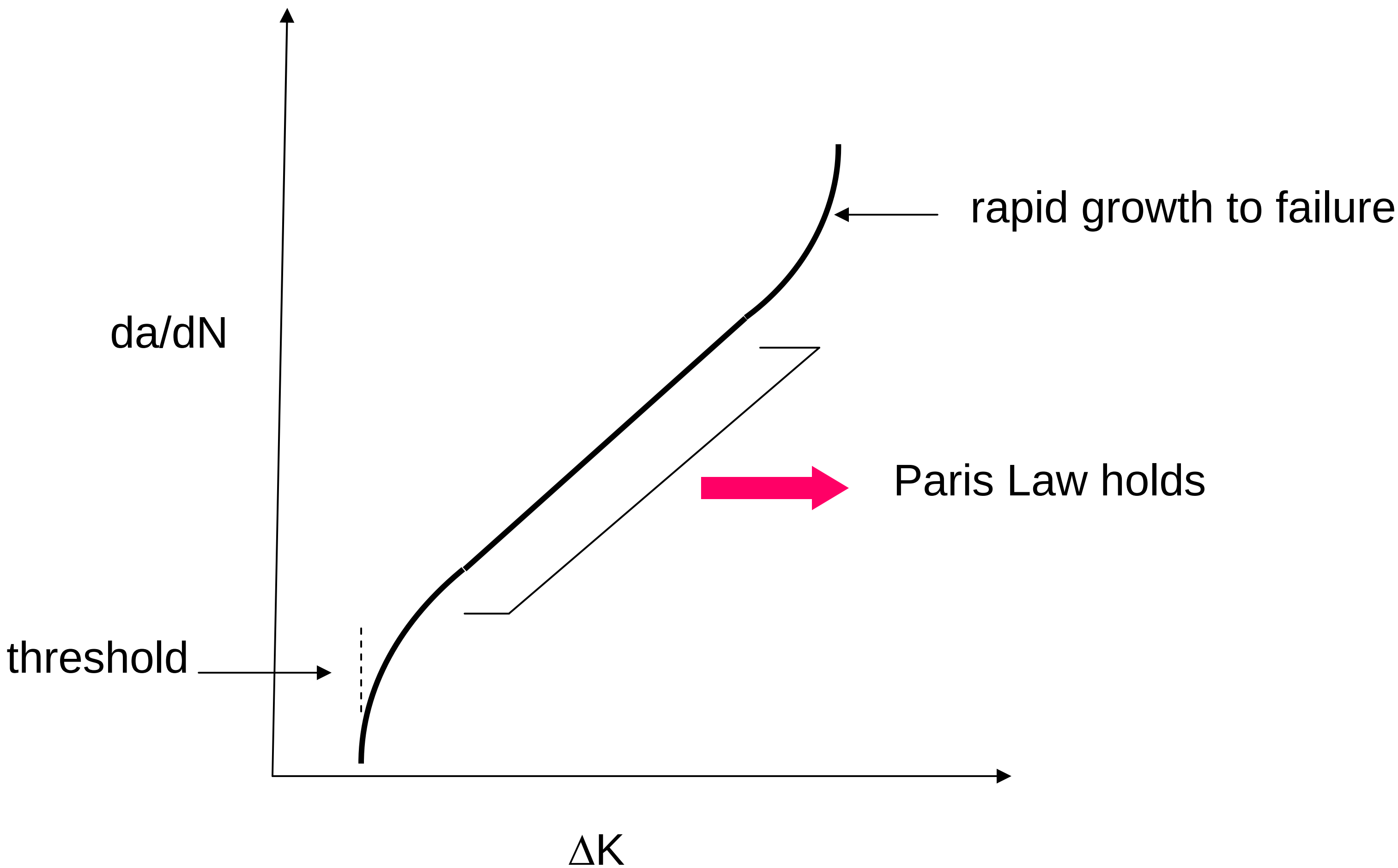
$$\Delta K = K_{\max} - K_{\min}$$

C, m … material and environmental dependent constants

a, ... crack size

N … number of cycles of loading

A Typical da/dN versus ΔK curve (log-log plot)



In terms of the stress

$$\frac{da}{dN} = C\left[\sqrt{\pi a}\, f\left(g,a\right)\Delta\sigma\right]^m \qquad \Delta\sigma = \sigma_{\max} - \sigma_{\min}$$

which can be rearranged and integrated

$$\int_{N_1}^{N_c} dN = \int_{a_1}^{a_c} \frac{da}{C\left[\sqrt{\pi a}\, f\left(g,a\right)\Delta\sigma\right]^m}$$

a_1 = crack size at N_1 cycles (initial crack size)
a_c = critical crack size at failure after N_c cycles

so $$\Delta N = \int_{a_1}^{a_c} \frac{da}{C\left[\sqrt{\pi a}\, f\left(g,a\right)\Delta\sigma\right]^m}$$

↑ no. of cycles left to failure

$$\Delta N = \int_{a_1}^{a_c} \frac{da}{C\left[\sqrt{\pi a}\, f\left(g, a\right) \Delta\sigma\right]^m}$$

If we know the configuration factor (fracture mechanics) and critical flaw size (material tests) and can measure a_1 through an NDE technique, then we can estimate the remaining life ΔN

References on Fracture Mechanics

H. Tada, P.C. Paris, and G.R. Irwin, The Stress Analysis of Cracks Handbook, 1973

D.P. Rooke and D.J. Cartwright, Compendium of Stress Intensity Factors, 1975

G.C.M. Sih, Handbook of Stress Intensity Factors, 1973

D. Broek, Elementary Engineering Fracture Mechanics, 1982

J.F. Knott, Fundamentals of Fracture Mechanics, 1973

J.M. Barsom and S.T. Rolfe, Fracture and Fatigue Control in Structures, 2nd Ed., 1987

T.L. Anderson, Fracture Mechanics, 1991.

Additional References

Fatigue Design – Life Expectancy of Machine Parts, Eliahu Zahavi, CRC Press , N.Y., 1996

Structural Life Assessment Methods, Alan F. Liu, ASM International, 1998.

Fatigue and Fracture Mechanics of High Risk Parts, Bahram Farahmoud, Chapman and Hall, 1997