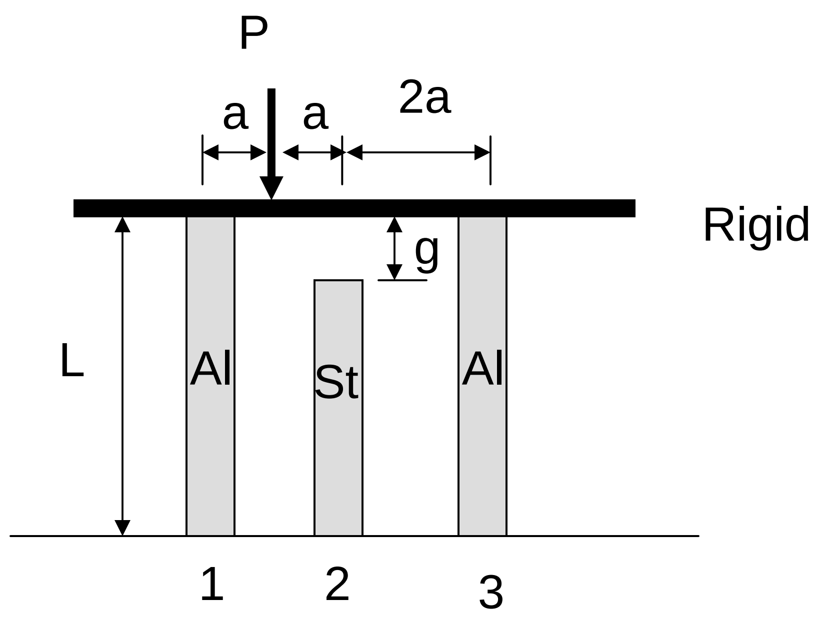


Two aluminum bars and a steel bar are loaded as shown. Assuming the load P closes the gap, g, determine the deformations and forces in the bars. The bars all have the same areas and $E_{st} = 3E_{Al}$

strain energy for a single bar

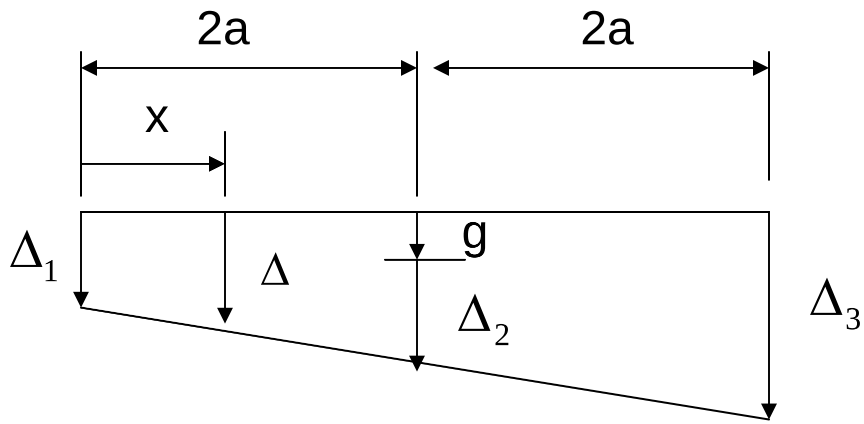
$$U = \frac{1}{2}\frac{EA}{L}\Delta^2 = \frac{1}{2}k\Delta^2$$

Δ = elongation

total strain energy

$$U = \frac{1}{2}k_{Al}\Delta_1^2 + \frac{1}{2}k_{St}\Delta_2^2 + \frac{1}{2}k_{Al}\Delta_3^2$$

$$= \frac{1}{2}k_{Al}\Delta_1^2 + \frac{3}{2}k_{Al}\Delta_2^2 + \frac{1}{2}k_{Al}\Delta_3^2$$



$$\frac{\Delta - \Delta_1}{x} = \frac{\Delta_3 - \Delta_1}{4a}$$

$$\Rightarrow \quad \Delta = \frac{x(\Delta_3 - \Delta_1)}{4} + \Delta_1$$

At the steel bar x = 2a $\Delta = \frac{\Delta_1 + \Delta_3}{2} = \Delta_2 + g$

$$U(\Delta_1, \Delta_3) = \frac{1}{2} k_{Al} \Delta_1^2 + \frac{3}{2} k_{Al} \left[\frac{\Delta_1 + \Delta_3}{2} - g \right]^2 + \frac{1}{2} k_{Al} \Delta_3^2$$

At the load x = a $\Delta = \Delta_P = \frac{3\Delta_1}{4} + \frac{\Delta_3}{4}$

Principle of virtual work

$$\delta U = \delta W$$

$$\frac{\partial U}{\partial \Delta_1} \delta\Delta_1 + \frac{\partial U}{\partial \Delta_3} \delta\Delta_3 = P \delta\Delta_P = P \frac{\partial \Delta_P}{\partial \Delta_1} \delta\Delta_1 + P \frac{\partial \Delta_P}{\partial \Delta_3} \delta\Delta_3$$

for all $\delta\Delta_1, \delta\Delta_3$

$$\Rightarrow \quad \frac{\partial U}{\partial \Delta_1} = P \frac{\partial \Delta_P}{\partial \Delta_1}$$

$$\frac{\partial U}{\partial \Delta_3} = P \frac{\partial \Delta_P}{\partial \Delta_3}$$

$$U\left(\Delta_1,\Delta_3\right)=\frac{1}{2}k_{Al}\Delta_1^2+\frac{3}{2}k_{Al}\left[\frac{\Delta_1+\Delta_3}{2}-g\right]^2+\frac{1}{2}k_{Al}\Delta_3^2$$

$$\frac{\partial U}{\partial \Delta_1}=P\frac{\partial \Delta_P}{\partial \Delta_1} \qquad \Delta=\Delta_P=\frac{3\Delta_1}{4}+\frac{\Delta_3}{4}$$

$$\frac{\partial U}{\partial \Delta_3}=P\frac{\partial \Delta_P}{\partial \Delta_3}$$

⇒

$$k_{Al}\Delta_1+3k_{Al}\left(\frac{\Delta_1+\Delta_3}{2}-g\right)\left(\frac{1}{2}\right)=\frac{3P}{4}$$

$$k_{Al}\Delta_3+3k_{Al}\left(\frac{\Delta_1+\Delta_3}{2}-g\right)\left(\frac{1}{2}\right)=\frac{P}{4}$$

or, equivalently

$$\frac{7\Delta_1}{4}+\frac{3\Delta_3}{4}=\frac{3P}{4k_{Al}}+\frac{3g}{2}$$

$$\frac{3\Delta_1}{4}+\frac{7\Delta_3}{4}=\frac{P}{4k_{Al}}+\frac{3g}{2}$$

Solving, we find

$$\Delta_1 = \frac{0.45P}{k_{Al}} + 0.6g$$

$$\Delta_3 = \frac{-0.05P}{k_{Al}} + 0.6g$$

(Compressional) forces in the bars are

$$P_1 = k_{Al}\Delta_1 = 0.45P + 0.6k_{Al}g$$

$$P_2 = 3k_{Al}\left[\frac{\Delta_1 + \Delta_3}{2} - g\right] = 0.6P - 1.2k_{Al}g$$

$$P_3 = k_{Al}\Delta_3 = -0.05P + 0.6k_{Al}g$$

Note: for $P_2 > 0$ we need $P > 2k_{Al}g$

Otherwise the gap will not be closed