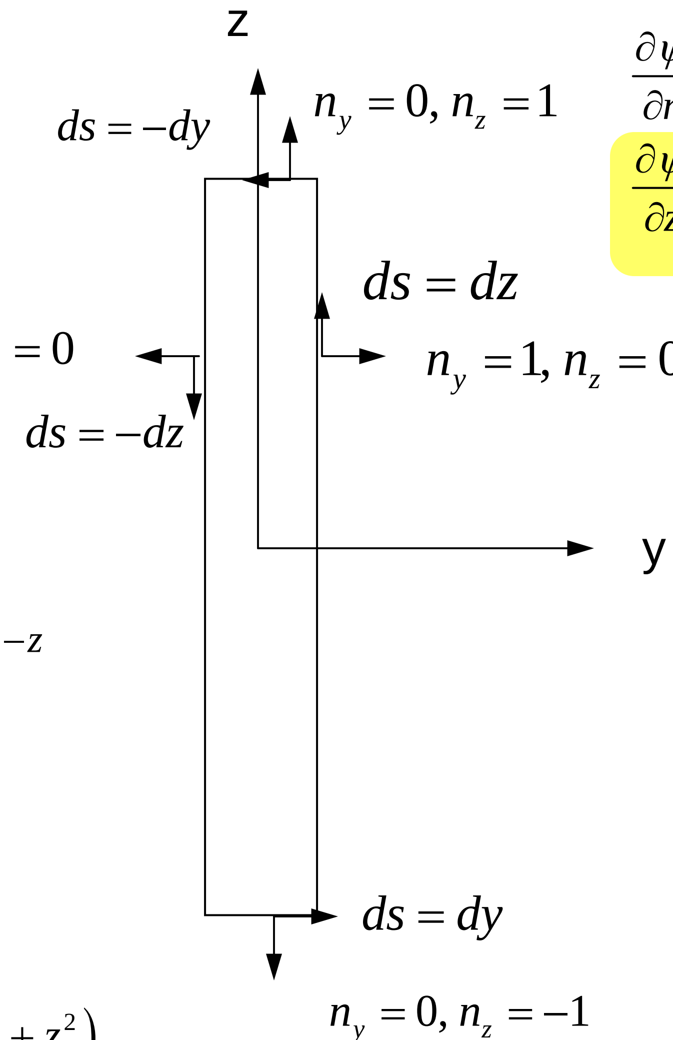
Torsion of a Thin Rectangular Section



$$\frac{\partial \psi}{\partial n} = \frac{1}{2}\frac{d}{ds}\left(y^2 + z^2\right)$$

$$\frac{\partial \psi}{\partial z} = \frac{1}{2}\frac{d}{-dy}\left(y^2 + z^2\right) = -y$$

$$\frac{\partial \psi}{\partial n} = \frac{1}{2}\frac{d}{ds}\left(y^2 + z^2\right)$$

$$\frac{\partial \psi}{\partial y} = \frac{1}{2}\frac{d}{dz}\left(y^2 + z^2\right) = z$$

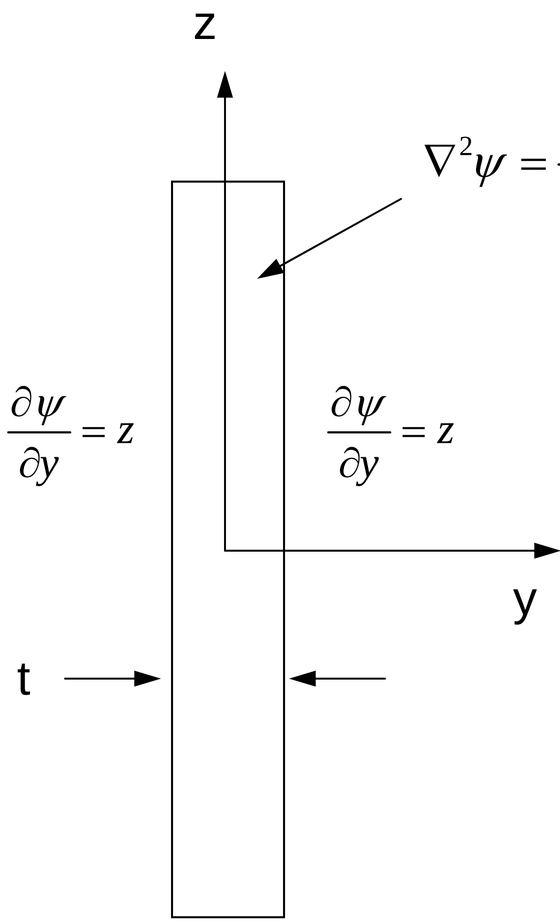
$$\frac{\partial \psi}{\partial n} = \frac{1}{2}\frac{d}{ds}\left(y^2 + z^2\right)$$

$$\frac{\partial \psi}{-\partial y} = \frac{1}{2}\frac{d}{-dz}\left(y^2 + z^2\right) = -z$$

$$\frac{\partial \psi}{\partial y} = z$$

$$\frac{\partial \psi}{\partial n} = \frac{1}{2}\frac{d}{ds}\left(y^2 + z^2\right)$$

$$\frac{\partial \psi}{-\partial z} = \frac{1}{2}\frac{d}{dy}\left(y^2 + z^2\right) = y$$



$$\nabla^2\psi = \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = 0$$

$$\psi = yz$$

$$\frac{\partial\psi}{\partial y} = z \qquad \frac{\partial\psi}{\partial y} = z$$

$$\sigma_{xy} = G\phi'\left(\frac{\partial\psi}{\partial y} - z\right)$$

$$\sigma_{xz} = G\phi'\left(\frac{\partial\psi}{\partial z} + y\right)$$

$$\Rightarrow \quad \sigma_{xz} = 2G\phi' y = \frac{2Ty}{J_{eff}}$$

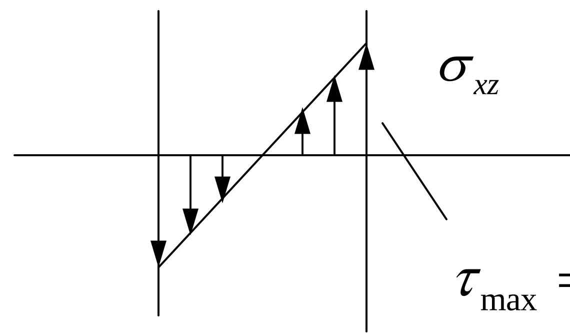
$$\sigma_{xy} = 0$$

$$T = GJ_{eff}\phi'$$

$$J_{eff} = \int_A \left\{\left(z - \frac{\partial\psi}{\partial y}\right)^2 + \left(y + \frac{\partial\psi}{\partial z}\right)^2\right\} dA$$

$$= 4\int_A y^2 dA = 4I_z = \frac{bt^3}{3}$$

$$J_{eff} = \frac{bt^3}{3}$$



$$\tau_{\max} = \frac{Tt}{J_{eff}}$$

Out of plane warping $u_x = \phi' yz$

```
>> z= linspace(-5,5, 20);
>> y =linspace(-0.5,0.5,10);
>> [zz, yy] = meshgrid(z,y);
>> ux=zz.*yy;
>> mesh(zz,yy,ux)
>> axis equal
>> view (50,20)
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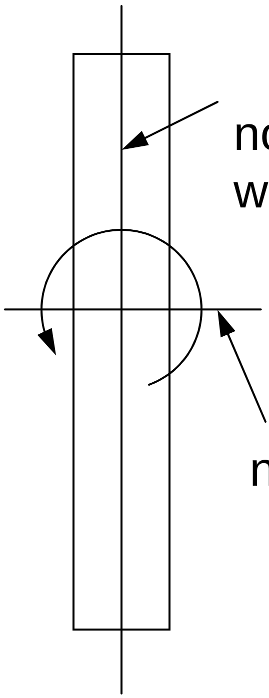
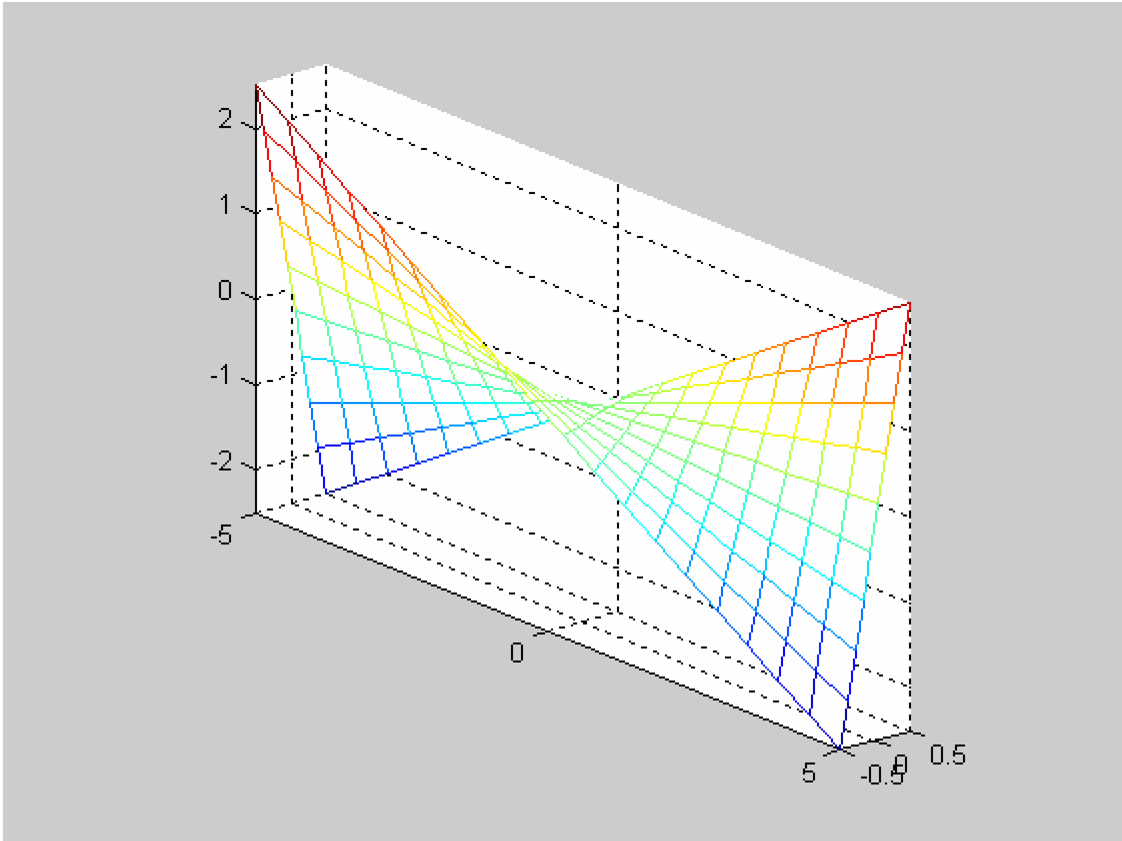
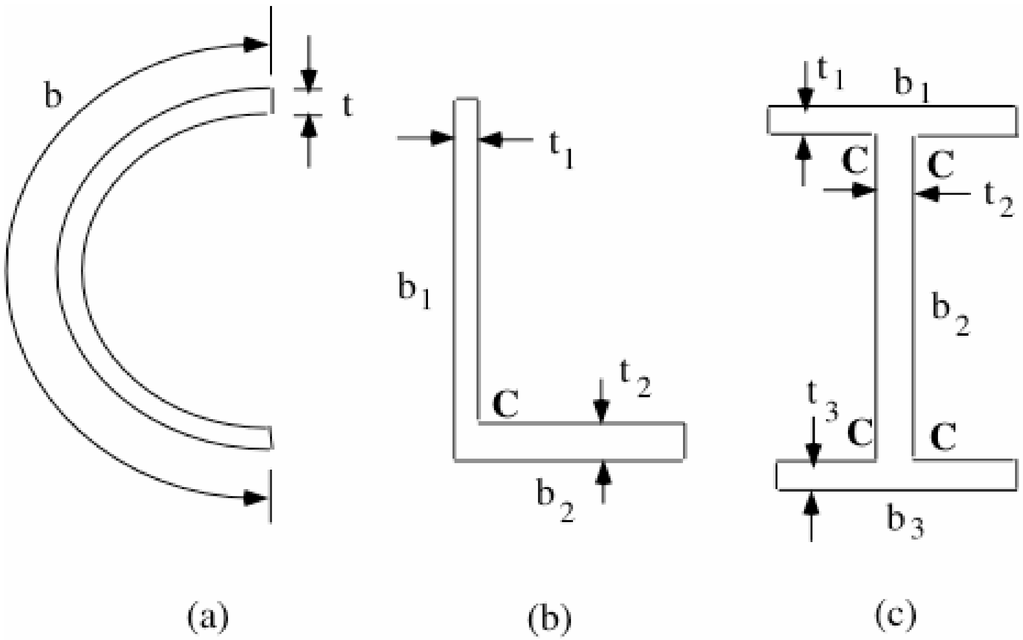




The results we obtained for the torsion of a thin rectangle can also be used, with some qualifications, for other thin open sections such as shown in the figure below



For example, the effective area moments for the cross sections shown can be calculated as

$$(a) \quad J_{eff} = \frac{1}{3} b t^3$$

$$(b) \quad J_{eff} = \frac{1}{3} b_1 t_1^3 + \frac{1}{3} b_2 t_2^3$$

$$(c) \quad J_{eff} = \frac{1}{3} b_1 t_1^3 + \frac{1}{3} b_2 t_2^3 + \frac{1}{3} b_3 t_3^3$$

Also, the maximum shear stress formula can still be applied as

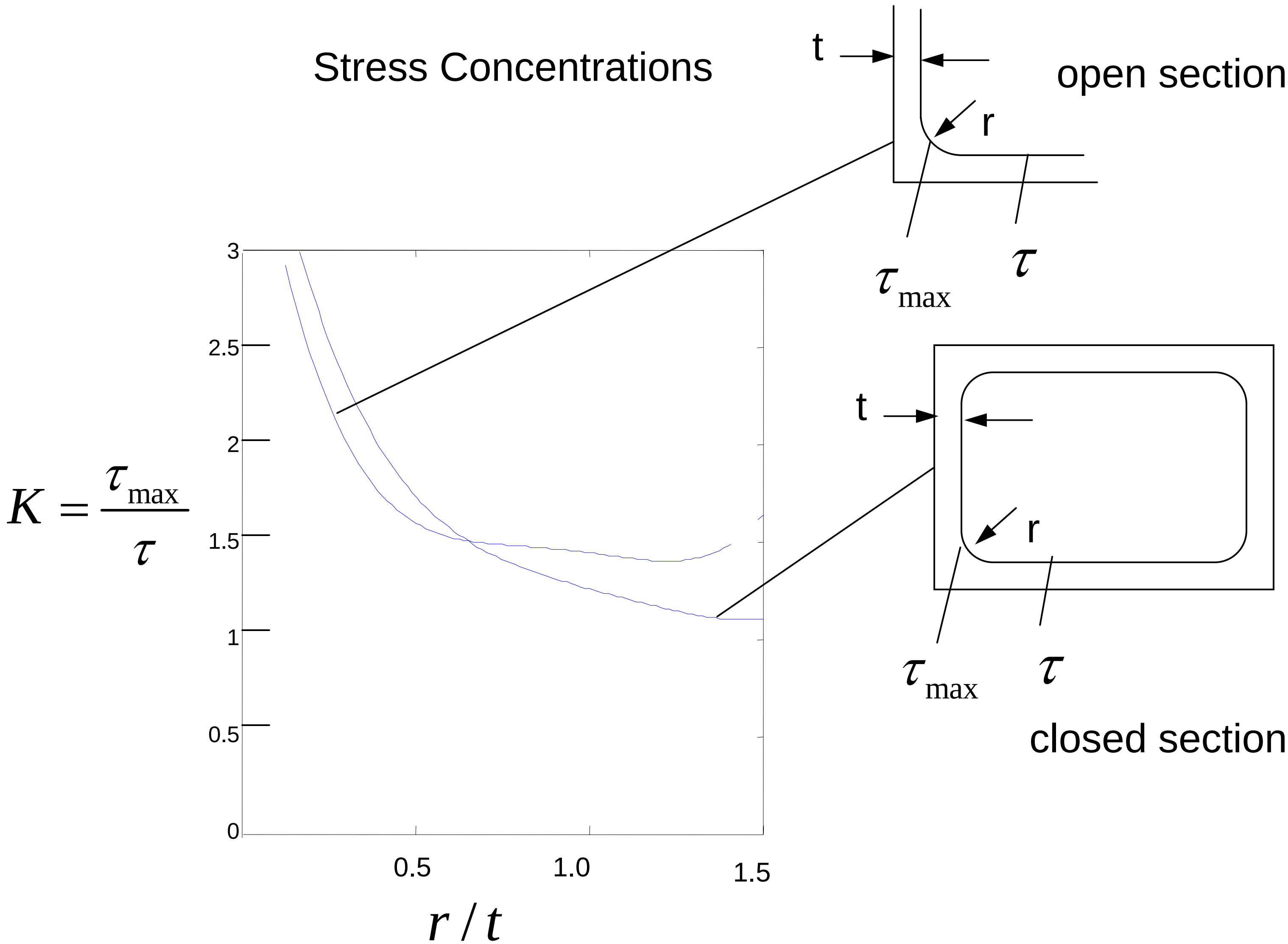
$$\tau_{\max} = \frac{Tt_{\max}}{J_{eff}}$$

where $t_{\max}$

is the largest thickness of the cross section. However, this maximum shear stress occurs on the outer edges of the thickest section and does not account for the stress concentrations that occur at re-entrant corners such as those marked with a **C** in Fig. 1. At such locations, the stresses depend on the local radius of curvature of the corner and may be considerably larger than the value predicted from Eq. (1). Such stress concentrations can be taken into account by finding either numerically or experimentally a stress concentration factor, *K*, for each re-entrant corner and then examining all high stress points and choosing the one with the highest stress, i.e.

$$\tau_{\max} = \left[K \frac{Tt}{J_{eff}} \right]_{\max}$$

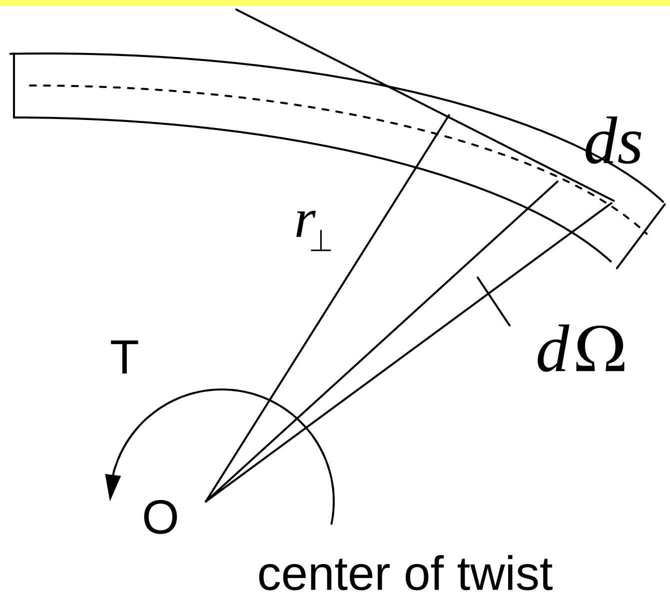
Stress Concentrations



Centerline warping of thin open sections

$$u_x = -\phi' \omega$$

$$\omega = \int r_\perp ds + \omega_0$$ sectorial area function



$$d\omega = r_\perp ds$$

$$d\Omega = \frac{1}{2} r_\perp ds$$

$$\omega = 2\int d\Omega = 2\Omega + \omega_0$$

since we are taking T as positive counterclockwise, ω is positive if the area is swept out in a counterclockwise manner

To locate the center of twist, O, we must have

$$\int y\omega dA = 0$$

$$\int z\omega dA = 0$$

y and z are measured from the centroid of the cross section

To fix ω_0 we can specify

$$\int \omega dA = o$$

An ω satisfying all three of the above conditions is called a principal sectorial area function, ω_p

$$u_x = -\phi' \, \omega_p$$