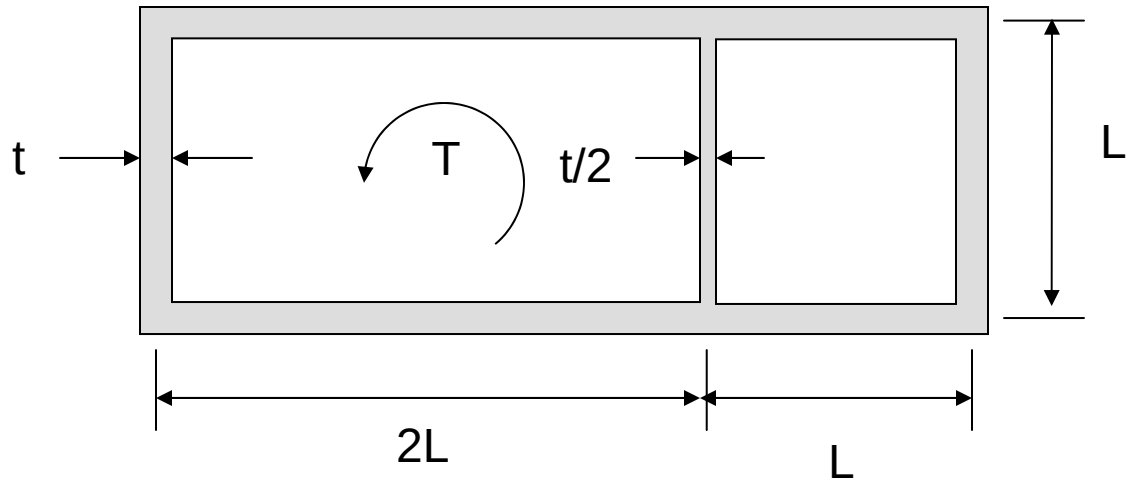
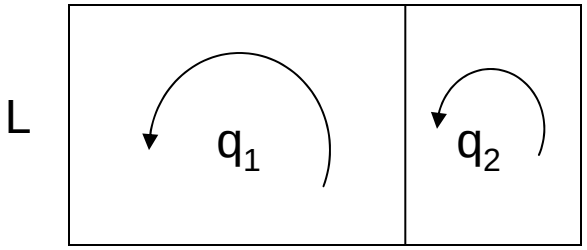
Determine the effective area moment for torsion of the two-cell box beam shown below. The thickness is t all around except for the vertical center section, whose thickness is t/2. All distances shown are measured from the center lines.





$$T = 2\sum q_i \Omega_i$$

$$\phi' = \frac{1}{2G\Omega_i} \oint_{C_i} \frac{q}{t} ds \quad (i = 1, 2)$$

$$T = 2\left[q_1 \left(2L^2 \right) + q_2 \left(L^2 \right) \right]$$

$$= L^2 \left[4q_1 + 2q_2 \right]$$

2L x L cell

$$\phi' = \frac{1}{2G\left(2L^2\right)t} \left[\frac{q_1 (5L)}{t} + \frac{(q_1 - q_2)(L)}{t/2} \right]$$

$$\Rightarrow \quad \frac{7}{4} q_1 - \frac{1}{2} q_2 = G\,\phi' Lt$$

L x L cell

$$\phi' = \frac{1}{2G\left(L^2\right)t} \left[\frac{q_2 (3L)}{t} + \frac{(q_2 - q_1)(L)}{t/2} \right]$$

$$\Rightarrow \quad \frac{5}{2} q_2 - q_1 = G\,\phi' Lt$$

$$\frac{7}{4}q_1 - \frac{1}{2}q_2 = G\,\phi' Lt$$

$$\frac{5}{2}q_2 - q_1 = G\,\phi' Lt$$

solving these equations simultaneously, we obtain $q_1 = \frac{24}{31}G\phi' Lt$

$$q_2 = \frac{22}{31}G\phi' Lt$$

so from $T = L^2\left[4q_1 + 2q_2\right]$

we obtain $T = \frac{140}{31}GL^3 t\phi'$

which gives $J_{eff} = \frac{140}{31}L^3 t$