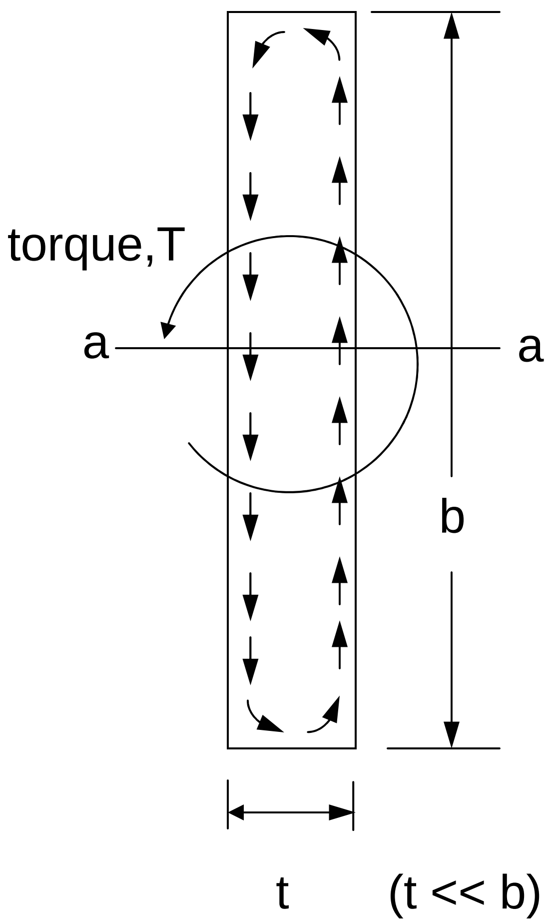
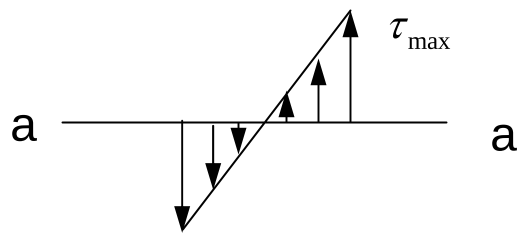
Torsion of thin, open sections



shear stress distribution



$$\tau_{\max} = \frac{Tt}{J_{eff}}$$

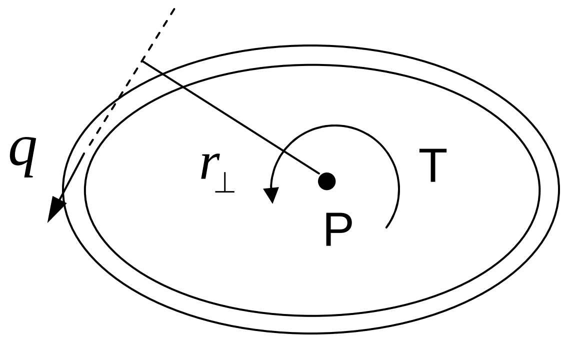
$$T = GJ_{eff}\phi'$$

twist/ unit length

$$J_{eff} = \frac{1}{3}bt^3$$

Torsion of a Thin Closed Section
(single cell)

Ω = area contained within the centerline of the cross section



$$T = \oint_C q r_\perp \, ds = q \oint_C r_\perp \, ds = 2q\Omega \quad \Rightarrow \quad q = \frac{T}{2\Omega} \qquad (1)$$

$$\phi' = \frac{1}{2G\Omega} \oint_C \frac{q}{t} \, ds = \frac{T}{4G\Omega^2} \oint_C \frac{ds}{t} \quad \Rightarrow \quad T = GJ_{eff}\,\phi' \qquad (2)$$

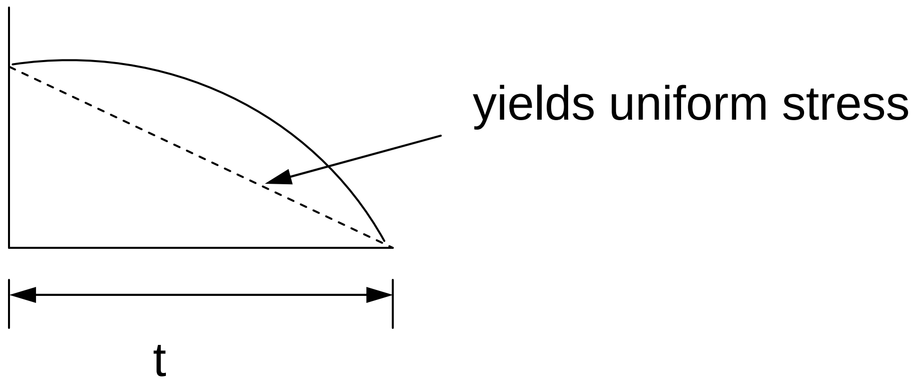
where $J_{eff} = \dfrac{4\Omega^2}{\oint_C \dfrac{ds}{t}}$

1. If T is known, q follows directly from Eq. (1), ϕ' is found from Eq.(2)

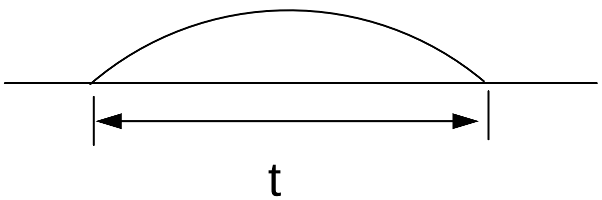
2. If ϕ' is known, T follows from Eq.(2), and q is then found from Eq. (1)

Torsion of a Thin Closed Section
(single cell)

The shear stress is not quite uniform across the thickness for thin closed sections



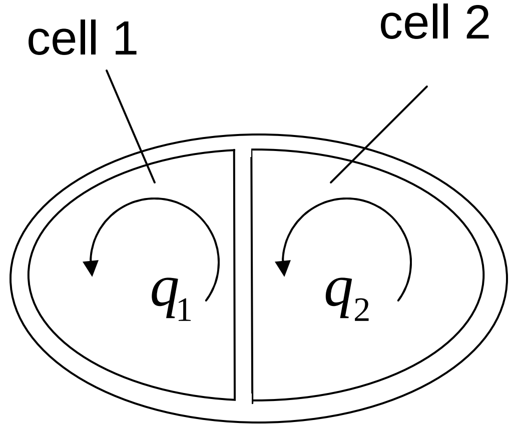
The difference looks much like that for an open section



so as a small correction factor:

$$J_{eff} = \frac{4\Omega^2}{\oint_C \frac{ds}{t}} + \frac{1}{3}\oint_C t^3(s)\,ds$$

Torsion of a Thin Closed Section
(multiple cells)



$$T = 2\Omega_1 q_1 + 2\Omega_2 q_2 \qquad (1)$$

$$\phi' = \frac{1}{2G\Omega_1} \oint_{C_1} \frac{q}{t}\, ds \qquad (2)$$

$$\phi' = \frac{1}{2G\Omega_2} \oint_{C_2} \frac{q}{t}\, ds \qquad (3)$$

(the q in Eqs.(2) and (3) is the <u>total q</u> flowing in a given cross section, i.e it is $q_1 - q_2$ flowing $\uparrow$ in the vertical section)

1. If the torque T is known, then q_1 and q_2 are first found in terms of the unknown ϕ' from Eqs. (2) and (3). These q_m 's are then placed into Eq.(1) which is solved for the unknown ϕ' . Once ϕ' is known in this manner, the q_m 's are completely determined.

2. If ϕ' is known, Eqs.(2) and (3) can be solved directly for the q_m 's and then Eq.(1) can be used to find the torque, T