Torsion of thin, open sections

\[ \tau = \frac{3Tt}{J} \]

\[ \tau_{\text{max}} = \frac{Tt}{J_{\text{eff}}} \]

\[ T = GJ_{\text{eff}} \phi' \]

\[ J_{\text{eff}} = \frac{1}{3}bt^3 \]

torque, \( T \)

t (t \ll b)
Torsion of a Thin Closed Section
(single cell)

\[ \Omega = \text{area contained within the centerline of the cross section} \]

\[ T = \oint_C q r_\perp ds \]
\[ = q \oint_C r_\perp ds = 2q\Omega \]
\[ \phi' = \frac{1}{2G\Omega} \oint_C \frac{q}{t} ds \]
\[ = \frac{T}{4G\Omega^2} \oint_C \frac{ds}{t} \]
\[ \Rightarrow T = GJ_{\text{eff}} \phi' \quad (2) \]

1. If \( T \) is known, \( q \) follows directly from Eq. (1), \( \phi' \) is found from Eq. (2)

2. If \( \phi' \) is known, \( T \) follows from Eq. (2), and \( q \) is then found from Eq. (1)
Torsion of a Thin Closed Section
(single cell)

The shear stress is not quite uniform across the thickness for thin closed sections.

The difference looks much like that for an open section

so as a small correction factor:

\[ J_{\text{eff}} = \frac{4\Omega^2}{\oint_c ds} + \frac{1}{3} \oint_c t^3(s) ds \]
1. If the torque \( T \) is known, then \( q_1 \) and \( q_2 \) are first found in terms of the unknown \( \phi' \) from Eqs. (2) and (3). These \( q_m \)'s are then placed into Eq.(1) which is solved for the unknown \( \phi' \). Once \( \phi' \) is known in this manner, the \( q_m \)'s are completely determined.

2. If \( \phi' \) is known, Eqs.(2) and (3) can be solved directly for the \( q_m \)'s and then Eq.(1) can be used to find the torque, \( T \).