Problem 3

```
>> stress = [ 40  40  30; 40 20  0; 30  0 20]
stress =
   40    40    30
   40    20     0
   30     0    20
```

stress state $\left[\sigma\right]$

```
>> n = [ cos(40*pi/180)  cos(75*pi/180)  cos(54*pi/180)]

n = 0.7660    0.2588    0.5878
```

$\{n\} = \{n_x \; n_y \; n_z\}$

unit normal to the plane (magnitude is not quite 1.0 exactly because angles given are only approximate)

```
>> norm(n)

ans = 0.9997
```

```
>> snn = n*stress*n'

snn = 74.6001
```

normal stress $\sigma_{nn} = \{n\}\left[\sigma\right]\{n\}^T$

```
>> tract = n*stress

tract = 58.6281   35.8182   34.7370
```

traction (stress) vector $\left\{T_x^{(n)} \; T_y^{(n)} \; T_z^{(n)}\right\} = \{n\}\left[\sigma\right]$

```
>> totshear = sqrt(norm(tract)^2 - snn^2)

totshear = 19.0180
```

total shear stress $\tau_s = \sqrt{\left|\mathbf{T}^{(n)}\right|^2 - \sigma_{nn}^2}$

```
>> sheardir = (tract - snn*n)/norm(totshear)

sheardir =   0.0779    0.8681   -0.4791
```

total shear stress direction

$$\mathbf{s} = \frac{\mathbf{T}^{(n)} - \sigma_{nn}\mathbf{n}}{\left|\tau_s\right|}$$

Problem 4

```
stress =
   12     6     9
    6    10     3
    9     3    14
```

original state of stress $[\sigma]$

```
>> [pdirs, pvals] = eig(stress)

pdirs =
  -0.7585   -0.0810    0.6467
   0.4244   -0.8144    0.3958
   0.4946    0.5746    0.6521

pvals =
   2.7734         0         0
        0    8.4798         0
        0         0   24.7467
```

state of stress in principal coordinates $[\sigma_p]$

```
>> noct = [ 1/sqrt(3)  1/sqrt(3)   1/sqrt(3)]

noct =  0.5774    0.5774    0.5774
```

unit normal components of the octahedral plane relative to the principal directions $\{n_{oct}\}$

```
>> norm_oct = noct*pvals*noct'

norm_oct = 12.0000
```

normal stress on the octahedral plane

$$(\sigma_{nn})_{oct} = \{n_{oct}\}[\sigma_p]\{n_{oct}\}^T$$

```
>> tract_oct = noct*pvals

tract_oct = 1.6012    4.8958   14.2875
```

traction (stress) vector on the octahedral plane

$$\left\{T_{p1}^{(n)}\ T_{p2}^{(n)}\ T_{p3}^{(n)}\right\} = \left\{n_{oct}\right\}\left[\sigma_p\right]$$

components of traction along principal directions

```
>> tau_oct = sqrt(norm(tract_oct)^2 - norm_oct^2)

tau_oct =  9.3095
```

total shear stress on the octahedral plane

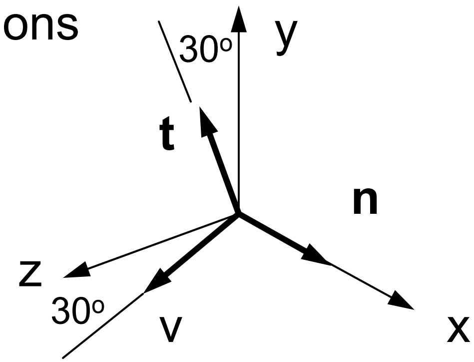
$$\left(\tau_s\right)_{oct} = \sqrt{\left|\mathbf{T}^{(n)}\right|^2 - \sigma_{nn}^2}$$

Problem 5

```
>> stress = [ 60  40  -40; 40  0  -20; -40  -20  20]
stress =
    60    40   -40
    40     0   -20
   -40   -20    20

>> n = [  1  0  0]

n =   1     0     0

>> t = [0  cos(30*pi/180)  sin(30*pi/180)]

t =  0    0.8660    0.5000

>> v = [ 0 -sin(30*pi/180)  cos(30*pi/180)]

v =  0   -0.5000    0.8660
```

original state of stress $[\sigma]$

unit vectors along the n, t, v directions




```
>> l = [ n'  t'  v']

l =

    1.0000         0         0
         0    0.8660   -0.5000
         0    0.5000    0.8660
```

form up the matrix of direction cosines

$$[l] = \begin{bmatrix} n_x & t_x & v_x \\ n_y & t_y & v_y \\ n_z & t_z & v_z \end{bmatrix}$$

```
>> new_stress = l'*stress*l

new_stress =

   60.0000   14.6410  -54.6410
   14.6410  -12.3205   -1.3397
  -54.6410   -1.3397   32.3205
```

compute the new state of stress along the n, t, v axes

$$[\sigma'] = [l]^T [\sigma][l]$$

Problem 6

```
>> stress = [ 12  4  2; 4  -8  -1; 2  -1  6]
stress =
    12     4     2
     4    -8    -1
     2    -1     6

>> [pdirs, pvals] =eig(stress)

pdirs =
   -0.1957    0.2204   -0.9556
    0.9764    0.1350   -0.1688
    0.0918   -0.9660   -0.2416

pvals =
   -8.8958         0         0
         0    5.6835         0
         0         0   13.2122


>> tau_max = (pvals(3,3)  -pvals(1,1))/2

tau_max =  11.0540


>> snn = (pvals(1,1) + pvals(3,3))/2

snn =   2.1582
```

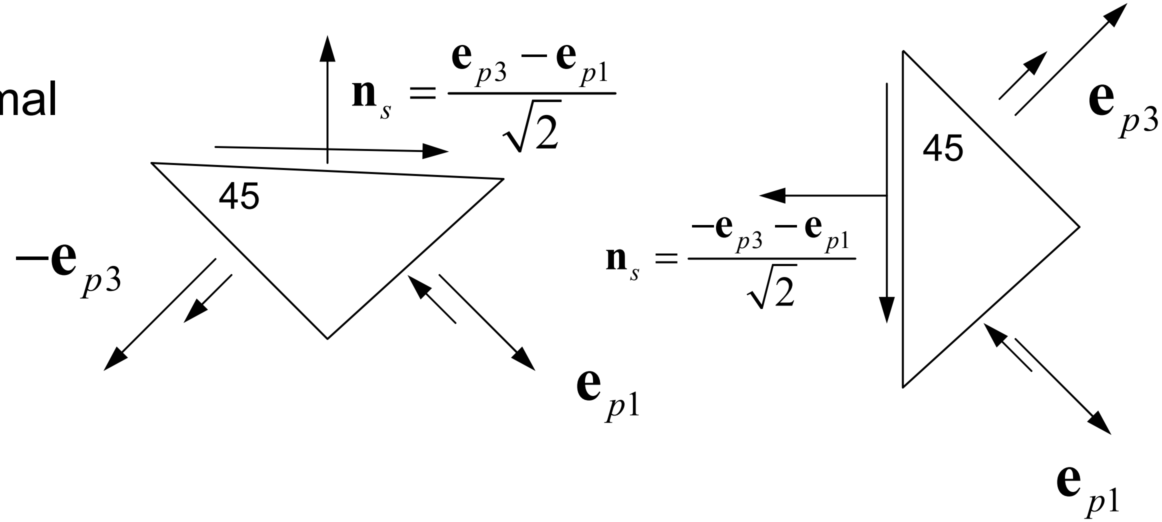
original state of stress $\left[\sigma\right]$

principal stresses $\begin{bmatrix} \sigma_{p1} & 0 & 0 \\ 0 & \sigma_{p2} & 0 \\ 0 & 0 & \sigma_{p3} \end{bmatrix}$

$$\left|\tau_{\max}\right| = \left|\frac{\sigma_{p3} - \sigma_{p1}}{2}\right|$$

$$\sigma_{nn} = \frac{\sigma_{p1} + \sigma_{p3}}{2}$$

max shear plane normal




```
>> e1 = pdirs(:,1)

e1 =

  -0.1957
   0.9764
   0.0918

>> e3 = pdirs(:,3)

e3 =

  -0.9556
  -0.1688
  -0.2416

>> n = (e3 -e1)/sqrt(2)

n =

  -0.5373
  -0.8098
  -0.2358
```

```
>> tract = n'*stress

tract =

  -10.1584    4.5645   -1.6794

>> snn=tract*n

snn =

    2.1582
```

← normal stress on max shear plane

```
>> totshear=sqrt(norm(tract)^2 -snn^2)

totshear =

   11.0540
```

← max shear stress

direction of max total shear stress

```
>> shear_dir = (tract -snn*n')/totshear

shear_dir =

  -0.8141    0.5710   -0.1059
```

← $\mathbf{e}_s$

```
>> shear_dir*n

ans =

  3.2613e-016
```

← check that the vector is in the plane, i.e.

$$\mathbf{e}_s \cdot \mathbf{n}_s = 0$$