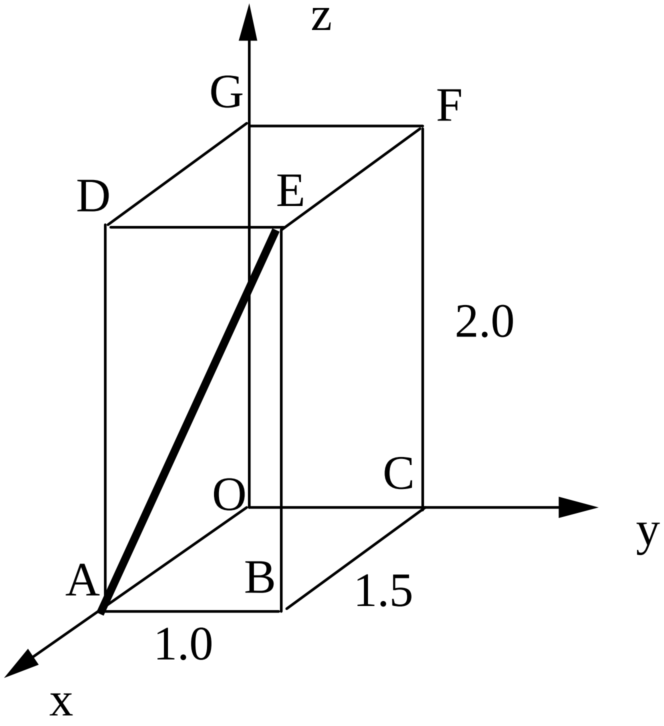
Problem 1 (a)



$$\mathbf{EA} = -2\mathbf{e}_z - 1\mathbf{e}_y, \quad |\mathbf{EA}| = \sqrt{5}$$

$$\mathbf{n}_{EA} = \frac{\mathbf{EA}}{|\mathbf{EA}|} = 0\mathbf{e}_x - 0.4472\mathbf{e}_y - 0.8944\mathbf{e}_z$$

$$
\begin{aligned}
e_{nn} &= e_{xx}n_x^2 + e_{yy}n_y^2 + e_{zz}n_z^2 \\
&+2(e_{xy}n_xn_y + e_{xz}n_xn_z + e_{zy}n_zn_y)
\end{aligned}
$$

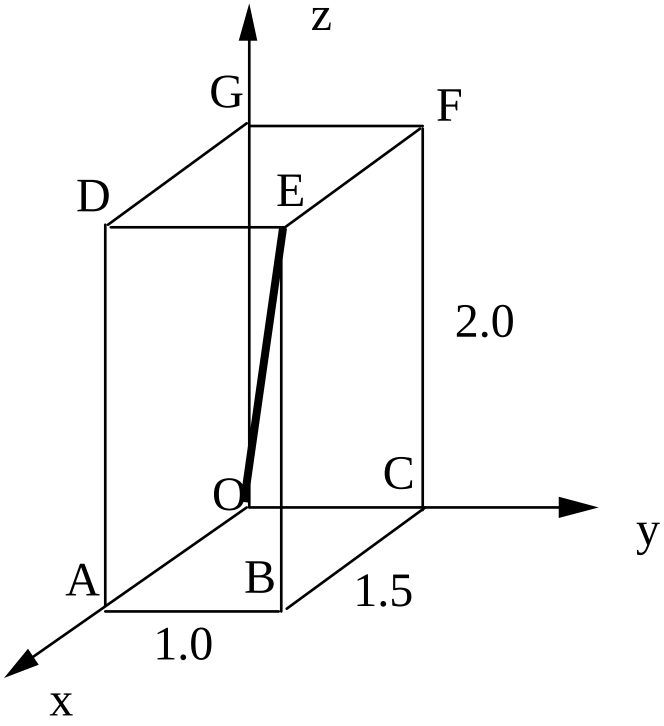
or, equivalently

$$
e_{nn} = \begin{Bmatrix} n_x & n_y & n_z \end{Bmatrix} \begin{bmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix}
$$

```
>> strain =[ 2670  2670  -330;
             2670  2000  -1500;
             -330 -1500  -2000]
strain =
        2670       2670       -330
        2670       2000      -1500
        -330      -1500      -2000
>> ea = [ 0  -1  -2]
ea =
     0    -1    -2
>> nea =ea/norm(ea)
nea =
         0   -0.4472   -0.8944
>> enn = nea*strain*nea'


enn =
       -2400
```

-2400μ

(b)



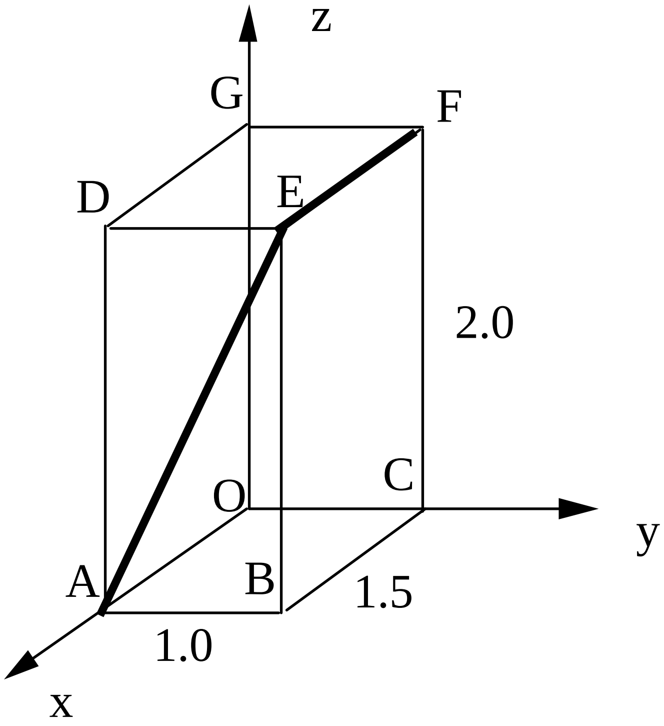
$$\mathbf{EO} = -1.5\mathbf{e}_x - 1\mathbf{e}_y - 2\mathbf{e}_z, \quad \left|\mathbf{EO}\right| = 2.6925$$

$$\mathbf{n}_{EO} = -0.557\mathbf{e}_x - 0.371\mathbf{e}_y - 0.743\mathbf{e}_z$$

```
>> eo=[ -1.5  -1  -2]

eo =

   -1.5000   -1.0000   -2.0000


>> neo =eo/norm(eo)

neo =

   -0.5571   -0.3714   -0.7428


>> enn = neo*strain*neo'

enn =

    5.1724
```

5μ

(c)



$$\mathbf{n}_{EA} = 0\mathbf{e}_x - 0.4472\mathbf{e}_y - 0.8944\mathbf{e}_z$$

$$\mathbf{t}_{EF} = -1\mathbf{e}_x$$

$$e_{nt} = e_{xx}n_x t_x + e_{yy}n_y t_y + e_{zz}n_z t_z$$
$$+e_{xy}\left(n_x t_y + n_y t_x\right)$$
$$+e_{xz}\left(n_x t_z + n_z t_x\right)$$
$$+e_{zy}\left(n_z t_y + n_y t_z\right)$$

or, equivalently

$$e_{nt} = \left\{t_x \quad t_y \quad t_z\right\}\begin{bmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{bmatrix}\begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix}$$

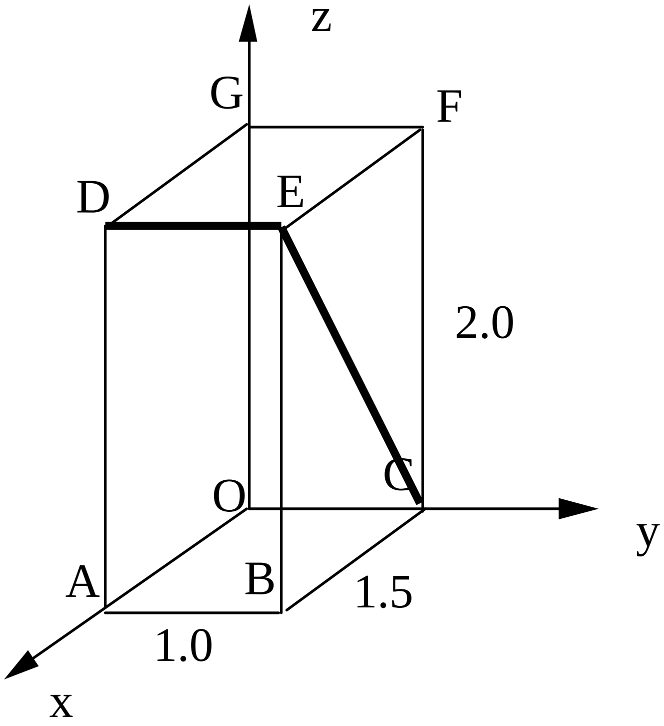
```
>> tef=[-1 0 0]

tef =

    -1     0     0

>> ent =tef*strain*nea'

ent =

  898.8993
```

tensor shear strain 900μ

```
>> engstrain = 2*ent

engstrain =

  1.7978e+003
```

engineering shear strain 1800μ

(d)



$$\mathbf{n}_{ED} = 0\mathbf{e}_x - 1\mathbf{e}_y + 0\mathbf{e}_z$$

$$\mathbf{EC} = -1.5\mathbf{e}_x - 2\mathbf{e}_z, \quad \left|\mathbf{EC}\right| = 2.5$$

$$\mathbf{t}_{EC} = -0.6\mathbf{e}_x + 0\mathbf{e}_y - 0.8\mathbf{e}_z$$

```
>> ned = [0  -1  0]

ned =

     0    -1     0


>> ec = [ -1.5  0  -2]

ec =

   -1.5000         0   -2.0000


>> tec = ec/norm(ec)

tec =

   -0.6000         0   -0.8000

>> ent =tec*strain*ned'

ent =

   402

>> engstrain = 2*ent

engstrain =

   804
```

$\underline{804\mu}$

(e) Principal strains

$$\begin{bmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix} = e_p \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix}$$

-2573μ
0.3μ
5243μ

(f) $\gamma_{\text{max}} = 5243 + 2573$

$= \underline{7816\mu}$

```
>> [pdirs, pvals] =eig(strain)

pdirs =

  0.1356   0.6807   0.7199
 -0.3794  -0.6356   0.6724
 -0.9152   0.3643  -0.1721
```

$\mathbf{e}_x$
$\mathbf{e}_y$
$\mathbf{e}_z$

```
pvals =

  1.0e+003 *


 -2.5729         0         0
       0    0.0003         0
       0         0    5.2426
```

Problem 2

$$[e]=\begin{bmatrix}300 & 50 & 0\\ 50 & 200 & 0\\ 0 & 0 & 100\end{bmatrix}\times 10^{-6}$$

(a) principal strains and principal strain directions

100μ
179.3μ
320.7μ

```
>> strain = [ 300  50  0;
              50  200 0;
               0   0 100]
strain =
   300    50     0
    50   200     0
     0     0   100
>> [pdirs, pvals] =eig(strain)
pdirs =
        0   0.3827   -0.9239
        0  -0.9239   -0.3827
     1.00        0         0
pvals =


  100.0000         0         0
         0  179.2893         0
         0         0  320.7107
```

$\mathbf{e}_x$
$\mathbf{e}_y$
$\mathbf{e}_z$

```
>> e =strm_to_strv(strain, 'strain')

e =

   300
   200
   100
   100
     0
     0
```

Convert strain matrix to strain vector

engineering shear strain

```
function y =strm_to_strv(input, type)
% strm_to_strv converts a 3x3 matrix of strains or stresses to a 6x1 vector
% first input argument is the 3x3 matrix
% the type argument is either 'strain' or 'stress'
% if type = 'strain' then the matrix shear strains are doubled to give engineering
% shear strains

if strcmp(type, 'stress')
    y(1)=input(1,1);
    y(2)=input(2,2);
    y(3) =input(3,3);
    y(4) = input(1,2);
    y(5) = input(1,3);
    y(6) = input(2,3);

elseif strcmp(type, 'strain')
    y(1)=input(1,1);
    y(2)=input(2,2);
    y(3) =input(3,3);
    y(4) = 2*input(1,2);
    y(5) = 2*input(1,3);
    y(6) = 2*input(2,3);

else
    error('wrong type')
end
% put in column vector
y=y';
```

```
>> c = [103  55   55  0     0    0;
          55  103  55  0     0    0;
          55  55  103  0     0    0;
           0    0    0  27.6  0    0;
           0    0    0    0 27.6   0;
           0    0    0    0    0  27.6]

c =

  103.0000   55.0000   55.0000         0         0         0
   55.0000  103.0000   55.0000         0         0         0
   55.0000   55.0000  103.0000         0         0         0
         0         0         0   27.6000         0         0
         0         0         0         0   27.6000         0
         0         0         0         0         0   27.6000
```

C-matrix in GPa

Compute stresses in MPa

```
>> s =(1.0E-03)*c*e
s =
   47.4000
   42.6000
   37.8000
    2.7600
         0
         0
```

Put stresses into 3x3 matrix

```
>> stress = strv_to_strm(s, 'stress')

stress =

   47.4000    2.7600         0
    2.7600   42.6000         0
         0         0   37.8000
```

(b) the stress components

```
function y=strv_to_strm(input, type)
% strv_to_strm converts a 6x1 vector of strains or stresses to a 3x3 matrix
% first input argument is the 6x1 vector
% the type argument is either 'strain' or 'stress'
% if type = 'strain' then the vector engineering shear strains are halved
%  to generate the tensor shear strains in the 3x3 matrix

if strcmp(type, 'stress')
    y(1,1) = input(1);
    y(2,2) = input(2);
    y(3,3) = input(3);
    y(1,2) = input(4);
    y(1,3) = input(5);
    y(2,3) = input(6);

elseif strcmp(type, 'strain')
    y(1,1) = input(1);
    y(2,2) = input(2);
    y(3,3) = input(3);
    y(1,2) = input(4)/2;
    y(1,3) = input(5)/2;
    y(2,3) = input(6)/2;

else
    error('wrong type')
end
y(2,1) =y(1,2);
y(3,1) = y(1,3);
y(3,2) = y(2,3);
```

(c) principal stresses

```
>> [pdirs2, pvals2] =eig(stress)

pdirs2 =
        0    0.4146   -0.9100
        0   -0.9100   -0.4146
   1.0000         0         0

pvals2 =
  37.8000         0         0
        0   41.3425         0
        0         0   48.6575
```

Principal stress directions (note differences from principal strain directions)

Principal stresses

```
pdirs =
        0    0.3827   -0.9239
        0   -0.9239   -0.3827
     1.00         0         0
```

principal strain directions

(d) E = 69 GPa, ν =0.33

$$\sigma_{xx} = \frac{E}{(1+\nu)(1-2\nu)}\left\{(1-\nu)e_{xx} + \nu\left(e_{yy} + e_{zz}\right)\right\}$$

$$\sigma_{yy} = \frac{E}{(1+\nu)(1-2\nu)}\left\{(1-\nu)e_{yy} + \nu\left(e_{xx} + e_{zz}\right)\right\}$$

$$\sigma_{zz} = \frac{E}{(1+\nu)(1-2\nu)}\left\{(1-\nu)e_{zz} + \nu\left(e_{xx} + e_{yy}\right)\right\}$$

$$\sigma_{xy} = G\gamma_{xy}, \sigma_{xz} = G\gamma_{xz}, \sigma_{zy} = G\gamma_{zy}$$

```
>> stress=stress_strain(69,0.33,strain)

stress =

   45.7762    2.5940         0

    2.5940   40.5882         0

         0         0   35.4003
```

$$\sigma_{xx} = 45.78\ MPa$$

$$\sigma_{yy} = 40.59\ MPa$$

$$\sigma_{zz} = 35.4\ MPa$$

$$\sigma_{xy} = 2.59\ MPa$$

```
function  stress = stress_strain(E, nu, strain)
% enter E in GPa, strain matrix in micro strain
%compute stress matrix in MPa
E = E*10^3; % put E in MPa
e =strain*10^(-6); % put micro factor in strain
C =E/((1+nu)*(1-2*nu));
G =E/(2*(1+nu));
stress=zeros(3,3);
stress(1,1) = C*((1-nu)*e(1,1) +nu*(e(2,2) +e(3,3)));
stress(2,2) = C*((1-nu)*e(2,2) +nu*(e(1,1) +e(3,3)));
stress(3,3) = C*((1-nu)*e(3,3) +nu*(e(1,1) +e(2,2)));
stress(1,2) =2*G*e(1,2);
stress(1,3) =2*G*e(1,3);
stress(2,3) =2*G*e(2,3);
%make stresses symmetric
stress(2,1) = stress(1,2);
stress(3,1) = stress(1,3);
stress(3,2) =stress(2,3);
```

principal stresses and principal stress directions

```
>> [pdirs3,pvals3]= eig(stress)

pdirs3 =


         0    0.3827   -0.9239        e_x

         0   -0.9239   -0.3827        e_y

      1.00         0         0        e_z

pvals3 =



  1.0e+007 *



    3.5400         0         0

         0    3.9514         0

         0         0    4.6851
```

$$\sigma_{p1} = 35.4\ MPa$$

$$\sigma_{p2} = 39.5\ MPa$$

$$\sigma_{p3} = 46.9\ MPa$$

Problem 3

stress state (MPa)

```
>> stress = [ 7  1.4  0; 1.4  2.1  0; 0  0 -2.8]

stress =
    7.0000    1.4000         0
    1.4000    2.1000         0
         0         0   -2.8000

>> [pdirs, pvals] =eig(stress)
```

principal stress directions

```
pdirs =
         0    0.2567   -0.9665
         0   -0.9665   -0.2567
    1.0000         0         0
```

principal stresses

```
pvals =

   -2.8000         0         0
         0    1.7282         0
         0         0    7.3718
```

d-matrix

```
>> d = [ 72.5  -36.25  -36.25    0    0       0;
        -36.25   942.5  -652.5    0    0       0;
        -36.25  -652.5   -1450    0    0       0;
             0      0           0  1015  0       0;
             0      0           0     0 1087.6  0;
             0      0           0     0     0    4350]

d =

  1.0e+003 *

    0.0725   -0.0362   -0.0362         0         0         0
   -0.0362    0.9425   -0.6525         0         0         0
   -0.0362   -0.6525   -1.4500         0         0         0
         0         0         0    1.0150         0         0
         0         0         0         0    1.0876         0
         0         0         0         0         0    4.3500
```

```
>> sigma = strm_to_strv(stress, 'stress')

sigma =

    7.0000
    2.1000
   -2.8000
    1.4000
         0
         0
```

put stresses in vector form

```
>> e = d*sigma

e =

  1.0e+003 *

    0.5329
    3.5525
    2.4360
    1.4210
         0
         0
```

calculate strains (μ strain)

```
>> strain = strv_to_strm(e, 'strain')

strain =

  1.0e+003 *

    0.5329    0.7105         0
    0.7105    3.5525         0
         0         0    2.4360

>> [pdirs2, pdvals2] = eig(strain)

pdirs2 =

   -0.9759         0    0.2182
    0.2182         0    0.9759
         0    1.0000         0


pdvals2 =

  1.0e+003 *

    0.3741         0         0
         0    2.4360         0
         0         0    3.7113
```

convert to strain matrix

state of strain

$$\begin{bmatrix} 532.9 & 710.5 & 0 \\ 710.5 & 3553 & 0 \\ 0 & 0 & 2436 \end{bmatrix}$$

principal strain directions

principal strains (μ)

347.1
2436
3711