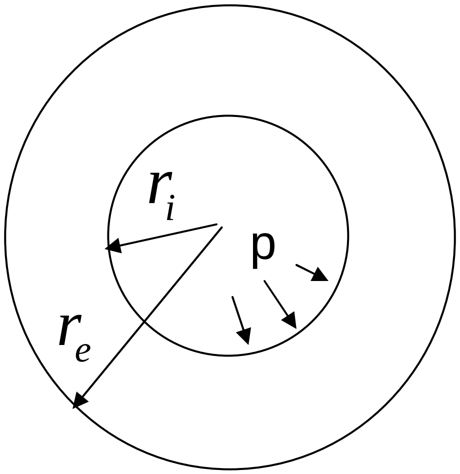
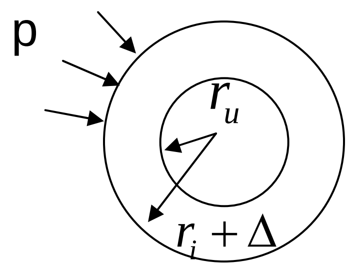
Shrink-fit Problems



cylinder 1



cylinder 2

Δ is called the interference

stresses are given by

$$\sigma_{rr} = A - \frac{B}{r^2}$$

$$\sigma_{\theta\theta} = A + \frac{B}{r^2}$$

$$A = \frac{p r_i^2}{r_e^2 - r_i^2}$$

$$B = \frac{p r_i^2 r_e^2}{r_e^2 - r_i^2}$$

$$\sigma_{rr} = A' - \frac{B'}{r^2}$$

$$\sigma_{\theta\theta} = A' + \frac{B'}{r^2}$$

$$A' = \frac{-p r_i^2}{r_i^2 - r_u^2}$$

$$B' = \frac{-p r_i^2 r_u^2}{r_i^2 - r_u^2}$$

The stresses do not involve the material properties but the displacements do

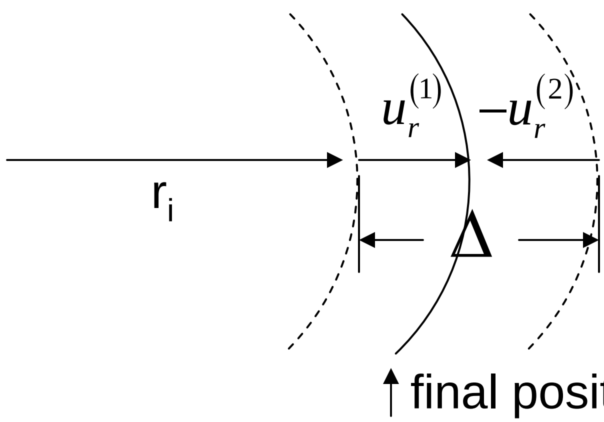
cylinder 1

$$u_r = \frac{1}{E_1}\left[\left(1+\nu_1\right)\frac{B}{r} + \left(1-\nu_1\right)Ar\right]$$

cylinder 2

$$u_r = \frac{1}{E_2}\left[\left(1+\nu_2\right)\frac{B'}{r} + \left(1-\nu_2\right)A'r\right]$$

At the interface r = r_i (actually at the radius somewhere between r_i , $r_i + \Delta$ but the interference is small so that we ignore this small difference)



$$u_r^{(1)} - u_r^{(2)} = \Delta \quad at\ r \cong r_i$$

↑ final position for interface

$$u_r^{(1)} - u_r^{(2)} = \Delta \quad at\ r \cong r_i$$

If $E_1 = E_2 = E$

$\nu_1 = \nu_2 = \nu$

$$\frac{p}{E}\left[\frac{(1+\nu)r_e^2 r_i^2 + (1-\nu)r_i^3}{r_e^r - r_i^2}\right] - \frac{p}{E}\left[\frac{-(1+\nu)r_u^2 r_i^2 - (1-\nu)r_i^3}{r_i^r - r_u^2}\right] = \Delta$$

Solving for Δ in terms of p

$$\Delta = \frac{p}{E}\left[\frac{2r_i^3\left(r_e^2 - r_u^2\right)}{\left(r_e^2 - r_i^2\right)\left(r_i^2 - r_u^2\right)}\right]$$

or vice-versa

$$p = E\Delta\left[\frac{\left(r_e^2 - r_i^2\right)\left(r_i^2 - r_u^2\right)}{2r_i^3\left(r_e^2 - r_u^2\right)}\right]$$

Example: Let

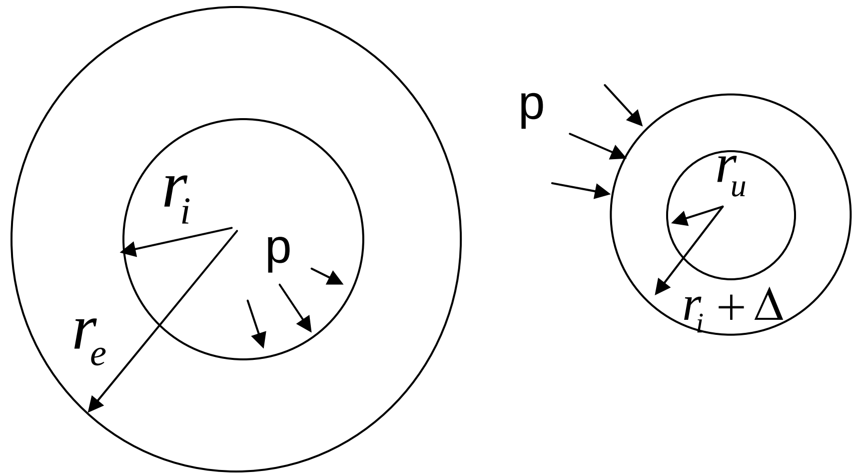
$$r_u = 50\ mm$$

$$r_i = 60\ mm$$

$$r_e = 70\ mm$$

$$\Delta = 1\ mm$$

$$E = 200\ GPa$$



$$p = E\Delta\left[\frac{\left(r_e^2 - r_i^2\right)\left(r_i^2 - r_u^2\right)}{2r_i^3\left(r_e^2 - r_u^2\right)}\right]$$

MPa

$$p = (1)\left(2x10^5\right)\frac{\left(70^2 - 60^2\right)\left(60^2 - 50^2\right)}{2\left(60^3\right)\left(70^2 - 50^2\right)}$$

$$= 276\ MPa$$

which is at or exceeding yield stress for steel