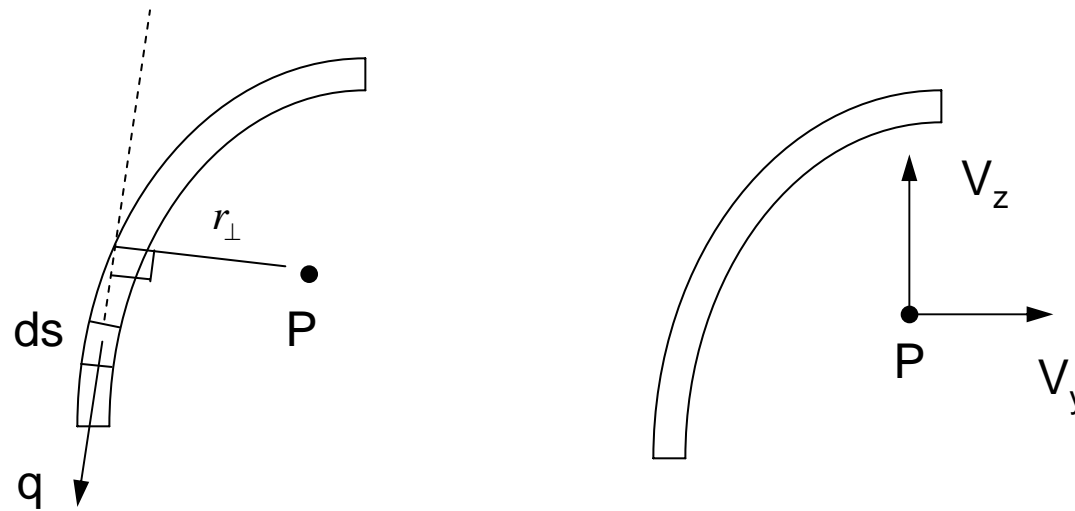


Determination of the location of the shear center through use of the sectorial area function



If the shear forces V_y and V_z act through the shear center, P , and hence produce only bending of the beam, then the shear flow, q , that produces those shear forces must not produce any moment about P , i.e.

$$\int_0^L q r_{\perp} ds = 0$$

However, since $q(0) = 0$, we have

$$q(s) = \frac{(V_z I_{yz} - V_y I_{yy}) Q_z + (V_y I_{yz} - V_z I_{zz}) Q_y}{I_{yy} I_{zz} - I_{yz}^2}$$

$$= F_z Q_z + F_y Q_y$$

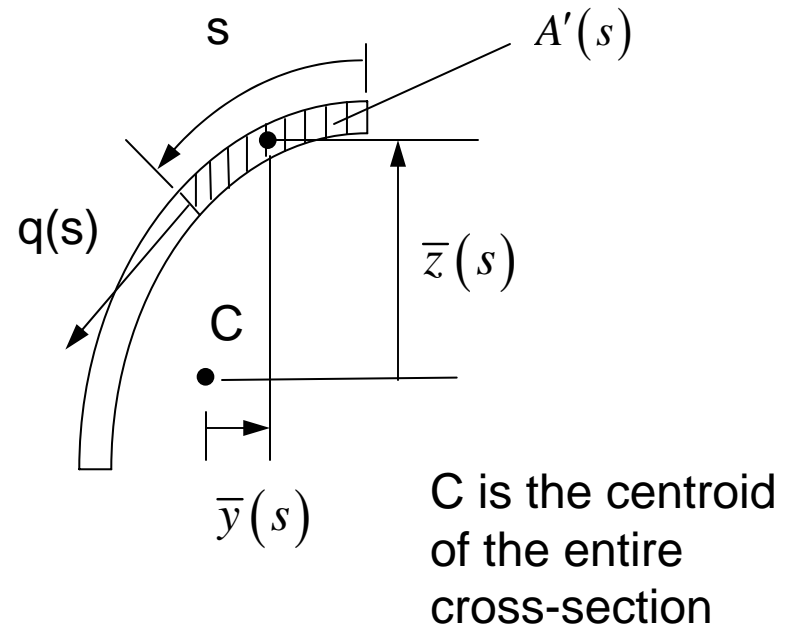
where

$$Q_z = \int_{A'(s)} y dA = \bar{y}(s) A'(s)$$

$$Q_y = \int_{A'(s)} z dA = \bar{z}(s) A'(s)$$

so

$$F_z \int_0^L r_{\perp} Q_z ds + F_y \int_0^L r_{\perp} Q_y ds = 0$$



\bar{y} and \bar{z} are measured from C to the centroid of A'

$$F_z \int_0^L r_{\perp} Q_z ds + F_y \int_0^L r_{\perp} Q_y ds = 0$$

Since this must be true for arbitrary F_z , F_y we must have

$$\int_0^L r_{\perp} Q_z ds = 0$$

$$\int_0^L r_{\perp} Q_y ds = 0$$

Consider

$$\int_0^L r_{\perp} Q_z ds = \int_0^L r_{\perp} \bar{y}(s) A'(s) ds = 0$$

Let $d\omega = r_{\perp} ds$

Then $r_{\perp} \bar{y}(s) A'(s) ds = \bar{y}(s) A'(s) d\omega = d[\bar{y}(s) A'(s) \omega] - \omega d[\bar{y}(s) A'(s)]$

So we have

$$\int_0^L r_{\perp} Q_z ds = \bar{y} A' \omega \Big|_{s=0}^{s=L} - \int_0^L \omega d[\bar{y} A'] = 0$$

But $\bar{y}(L) = 0, \quad A'(0) = 0$

so the first term on the right hand side of the above equation vanishes and we find $\int_0^L \omega d[\bar{y} A'] = 0$

Since $\bar{y} A' = \int_{A'} y dA$ it follows that $d[\bar{y} A'] = y dA$ and we obtain , finally

$$\int_0^L \omega y dA = 0$$

In an entirely similar fashion from $\int_0^L r_{\perp} Q_y ds = 0$ we can show

$$\int_0^L \omega z dA = 0$$

Summary:

We have shown that in order for the shear forces to produce only bending, we must have satisfied

$$\int_0^L \omega y dA = 0$$

$$\int_0^L \omega z dA = 0$$

ω is called the sectorial area function. We see it is given by

$$\omega(s) = \int_0^s r_{\perp} ds + \omega_0$$

↑
constant

To specify a value for the constant (which does not affect the above two conditions), we can also require

$$\int_0^L \omega dA = 0 \quad \left(\text{this gives what is called the } \underline{\text{principal sectorial area function}} \right)$$

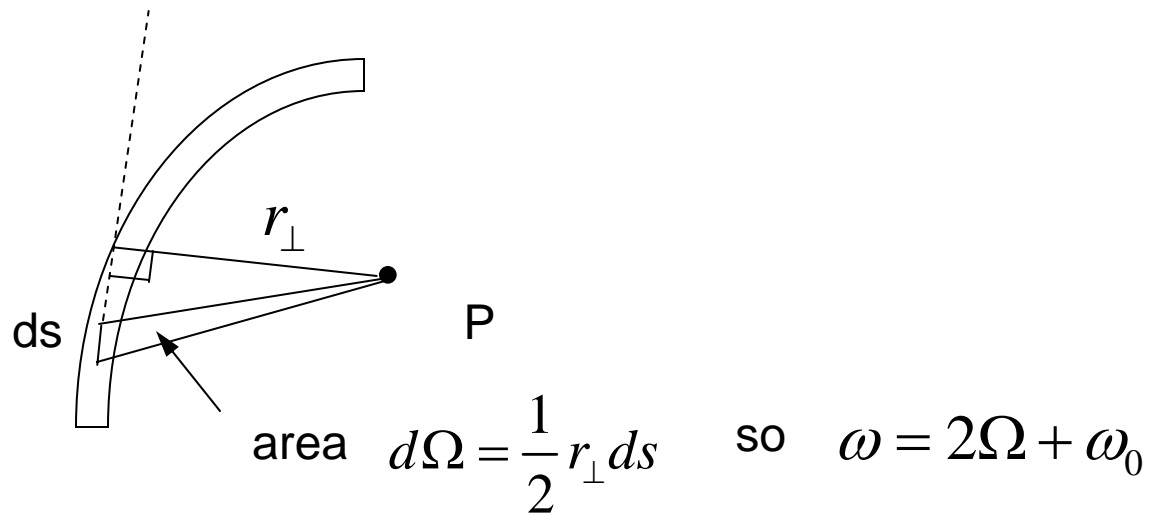
The principal sectorial area function satisfies

$$\int_0^L \omega y dA = 0$$

$$\int_0^L \omega z dA = 0$$

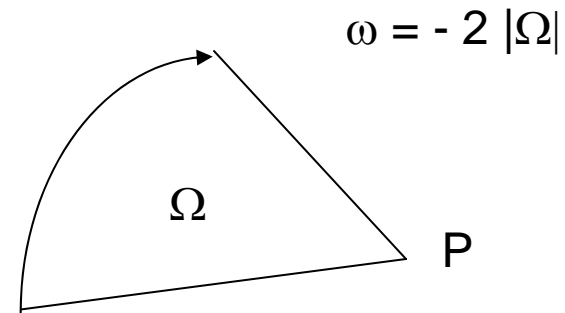
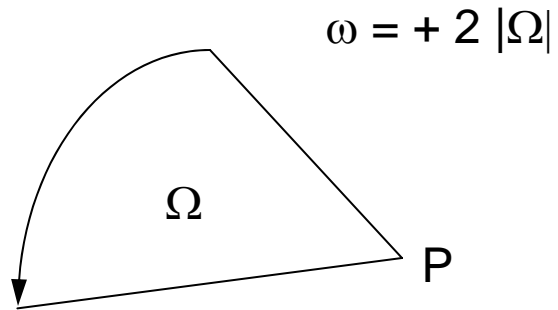
$$\int_0^L \omega dA = 0$$

Geometrical
interpretation of ω :



Thus, the sectorial area function is just twice the area swept out by the radius from P to the centerline of the cross-section (to within a constant).

Sign convention:



sectorial area function coming from area swept out counterclockwise is positive, coming from area swept out clockwise is negative