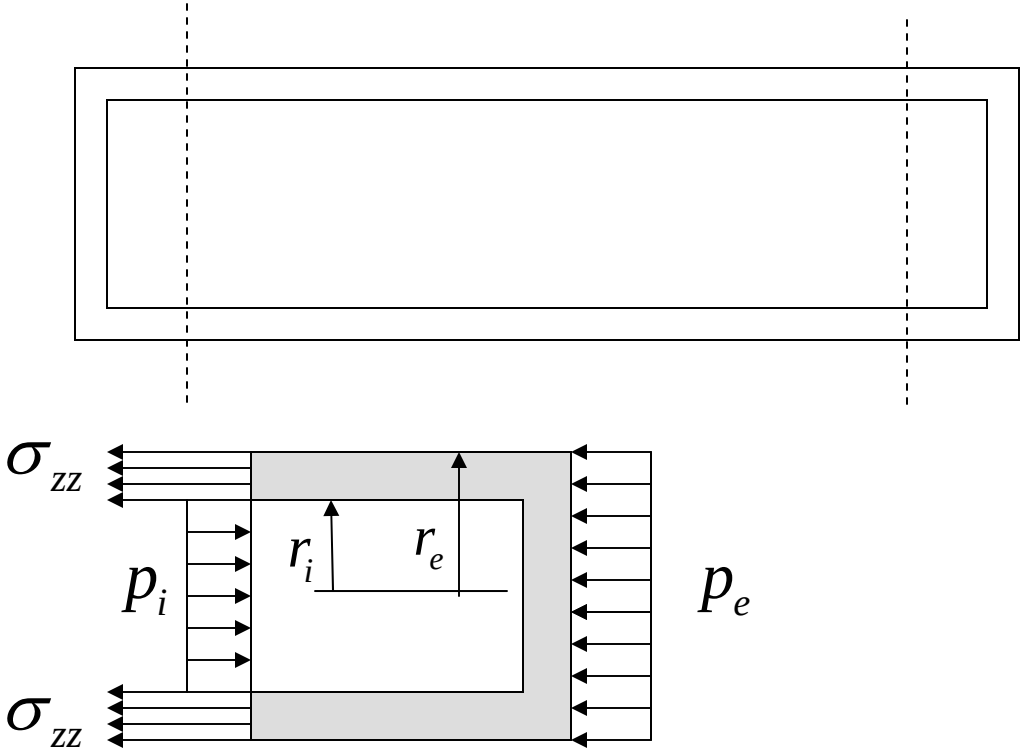
Thick-wall pressure vessel – closed ends



$$\sigma_{zz}\pi\left(r_e^2 - r_i^2\right) = p_i\pi r_i^2 - p_e\pi r_e^2$$

which gives

$$\sigma_{zz} = \frac{p_i r_i^2 - p_e r_e^2}{r_e^2 - r_i^2}$$

From Hooke's law $$\frac{u_r}{r} = e_{\theta\theta} = \frac{\sigma_{\theta\theta}}{E} - \frac{\nu\left(\sigma_{rr} + \sigma_{zz}\right)}{E}$$

so adding the axial stress σ_{zz} changes u_r by adding the term

$$\left(u_r\right)_{\sigma_{zz}} = \frac{-\nu\sigma_{zz} r}{E}$$

but the stresses σ_{rr} , $\sigma_{\theta\theta}$ are unaffected

The total axial strain is now

$$\begin{aligned} e_{zz} &= \left(e_{zz}\right)_{\sigma_{zz}} + \left(e_{zz}\right)_{\sigma_{rr},\sigma_{\theta\theta}} \\ &= \frac{\sigma_{zz}}{E} - \frac{2\nu}{E}\left(\frac{p_i r_i^2 - p_e r_e^2}{r_e^2 - r_i^2}\right) \\ &= \frac{\left(1-2\nu\right)}{E}\left(\frac{p_i r_i^2 - p_e r_e^2}{r_e^2 - r_i^2}\right) \end{aligned}$$