

Palmgren-Miner Rule

Suppose a body can tolerate only a certain amount of damage, D . If that body experiences damages D_i ($i=1, \dots, N$) from N sources, then we might expect that failure will occur if

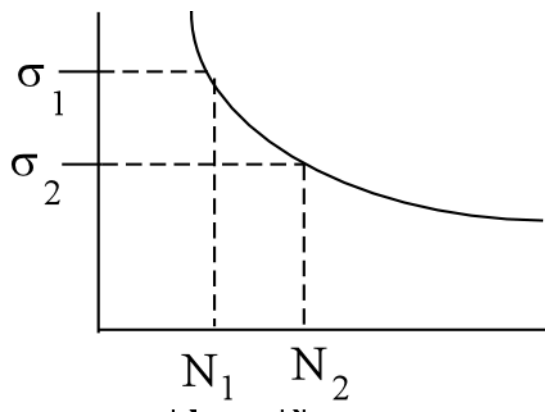
$$\sum_{i=1}^N D_i = D$$

or, equivalently

$$\sum_{i=1}^N \frac{D_i}{D} = 1$$

defines failure, where D_i/D is the fractional damage received from the i th source.

We can use this linear damage concept in a fatigue setting by considering the situation where a component is subjected to n_1 cycles at alternating stress σ_1 , n_2 cycles at stress σ_2 , ..., n_N cycles at σ_N . From the S-N curve for this material, then we can find the number of cycles to failure, N_1 at σ_1 , N_2 at σ_2 , ..., N_N at σ_N .



It is reasonable in this case to let the fractional damage at stress level σ_i be simply n_i/N_i , so that the Palmgren-Miner rule would say that fatigue failure occurs when

$$\sum_{i=1}^N \frac{n_i}{N_i} = 1$$

Example:

A part is subjected to a fatigue environment where 10% of its life is spent at an alternating stress level, σ_1 , 30% is spent at a level σ_2 , and 60% at a level σ_3 . How many cycles, n , can the part undergo before failure?

If, from the S-N diagram for this material the number of cycles to failure at σ_i is N_i ($i = 1, 2, 3$), then from the Palmgren-Miner rule failure occurs when:

$$\frac{0.1n}{N_1} + \frac{0.3n}{N_2} + \frac{0.6n}{N_3} = 1$$

so solving for n gives

$$n = \frac{1}{\left(\frac{0.1}{N_1} + \frac{0.3}{N_2} + \frac{0.6}{N_3} \right)}$$

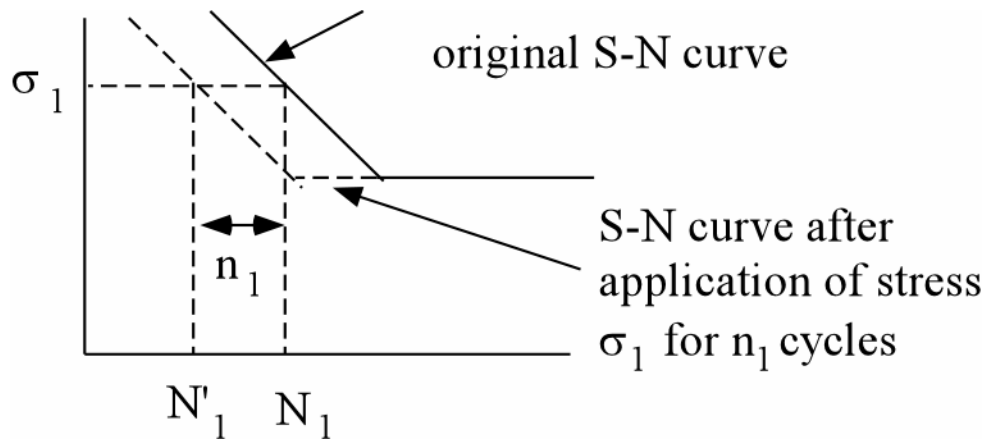
Remarks

1. "High-low" fatigue tests where testing occurs sequentially at two stress levels (σ_1, σ_2) where $\sigma_1 > \sigma_2$ generally shows that failure occurs when

$$\sum_{i=1}^2 \frac{n_i}{N_i} = c$$

where c normally is < 1 , i.e the Palmgren-Miner rule is non-conservative for these tests. For "low-high" tests, c values are typically > 1 .

2. For tests with random loading histories at several stress levels, correlation with the Palmgren-Miner rule is generally very good.
3. The Palmgren-Miner rule can be interpreted graphically as a "shift" of the S-N curve. For example, if n_1 cycles are applied at stress level σ_1 (where the life is N_1 cycles), the S-N curve is shifted so that goes through a new life value, N'_1 :



4. A major limitation of the Palmgren-Miner rule is that it does not consider sequence effects, i.e. the order of the loading makes no difference in this rule. Sequence effects are definitely observed in many cases. A second limitation is that the Palmgren-Miner rule says that the damage accumulation is independent of stress level. This can be seen from the modified S-N diagram above where the entire curve is shifted the same amount, regardless of stress amplitude.