The four pillars of all stress analyses:

- Equilibrium
- Compatibility
- Boundary conditions
- Stress-strain
equilibrium

\[ \sum F = 0 \implies \sum_{j=1}^{3} \frac{\partial \sigma_{ji}}{\partial x_j} = 0 \]

\[ \sum M = 0 \implies \sigma_{ij} = \sigma_{ji} \]

locally everywhere
compatibility

3-D: \( R_{ijkl} = 0 \)

2-D: \[
2 \left( \frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2} - \frac{\partial^2 \varepsilon_{11}}{\partial x_2^2} - \frac{\partial^2 \varepsilon_{22}}{\partial x_1^2} \right) = 0
\]

Given \( u(x) \) we can always find compatible strains through differentiation:

\[
\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

compatible strains \( \Rightarrow \) single valued displacements locally everywhere
stress-strain

For a linear elastic material

$$\begin{pmatrix} \sigma \end{pmatrix} = \begin{pmatrix} C \end{pmatrix} \begin{pmatrix} \varepsilon \end{pmatrix}$$
boundary conditions

\[ \sigma_{yy} = -p_o, \quad \sigma_{yx} = \sigma_{yz} = 0 \]

\[ T^{(n)} = 0 \]

\[ u = 0 \]