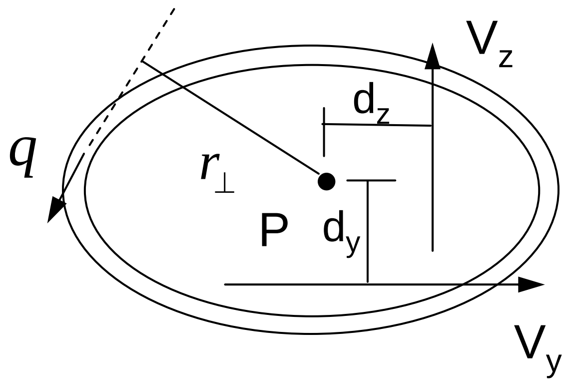
Bending and Torsion of a Thin Closed Section
(single cell)

Ω = area contained within the centerline of the cross section



$$\sum M_P = V_y d_y + V_z d_z = \oint_C q\, r_\perp ds$$

$$q(s) = q_c + V_y f(s) + V_z g(s)$$

this unknown constant is due to both bending and torsion

$$f(s) = \frac{I_{yz}Q_y - I_{yy}Q_z}{I_{yy}I_{zz} - I_{yz}^2}$$

$$g(s) = \frac{I_{yz}Q_z - I_{zz}Q_y}{I_{yy}I_{zz} - I_{yz}^2}$$

Placing the q expression into the moment equation gives:

$$\sum M_P = 2\Omega\, q_c + \oint_C \left[V_y f\left(s\right) + V_z g\left(s\right) \right] r_\perp ds \qquad (1)$$

Also, we have

$$\phi' = \frac{1}{2G\Omega} \oint_C \frac{q}{t} ds \qquad (2)$$

1. If the shear forces and their positions are known, then q_c can be found directly from Eq.(1) since the left hand side of that equation is known explicitly and f and g can be found for the given geometry. Then Eq. (2) can be used (since q is now given completely) to find ϕ'

2. If the shear forces are known but assumed to act through the shear center (whose position is unknown), we can set $\phi' = 0$, $V_y = 0$ and solve Eq.(2) for the unknown q_c . Then Eq.(1) gives the location of the shear center, d_z , since

$$V_z d_z = \oint_C q\, r_\perp ds$$

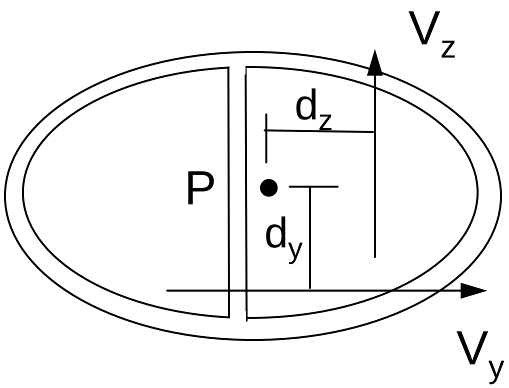
$$q(s) = q_c + V_z g(s)$$

We can repeat this process by setting $\phi' = 0$, $V_z = 0$ and solving Eq.(2) again for a new q_c . The Eq.(1) gives the location of the shear center, d_y :

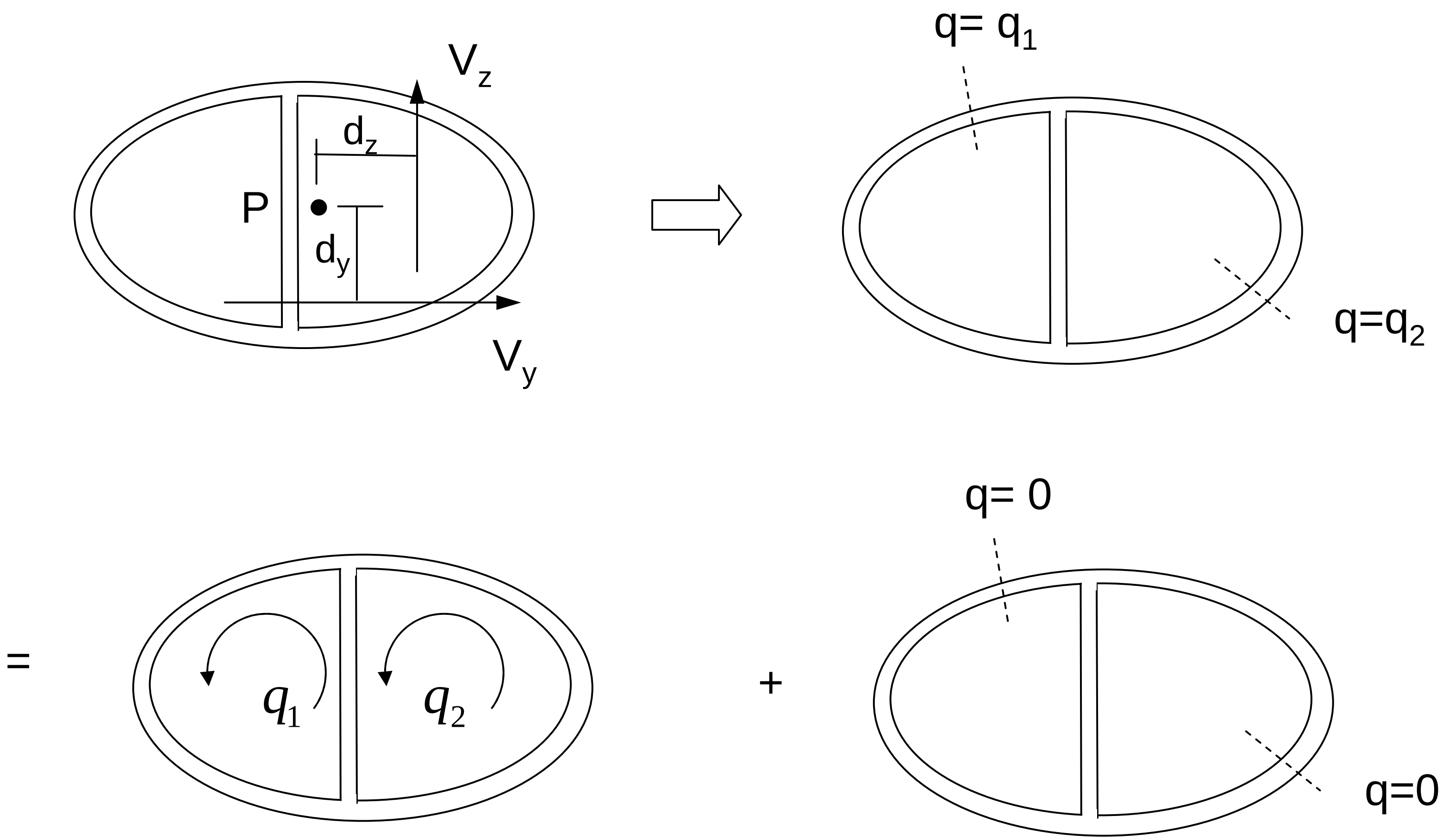
$$V_y d_y = \oint_C q\, r_\perp ds$$

$$q(s) = q_c + V_y f(s)$$

Bending and Torsion of a Thin Closed Section (multiple cells)

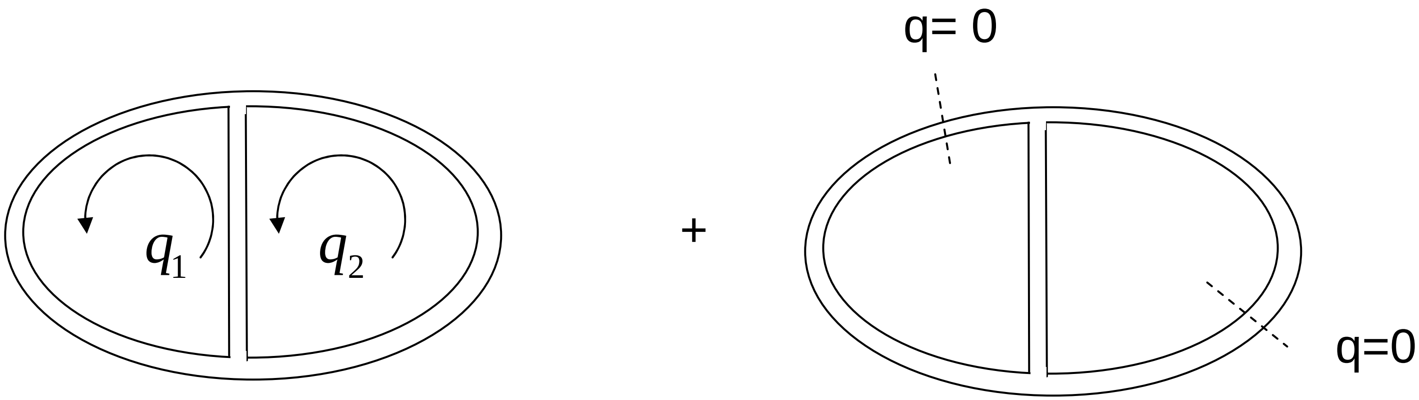


As in the case of the torsion of a multiple cell closed section, we need to account for unknown constant shear flows in each cell. This can be accomplished by conceptually decomposing our problem into two simpler problems as shown on the next page.



constant shear flows from constant parts of q due to bending and constant q's due to torsion.

bending shear flows q(s) due to this "open" section



$$V_y d_y + V_z d_z = 2\Omega_1 q_1 + 2\Omega_2 q_2 + \sum_{m=1}^{2} \oint_{C_m} \left[V_y f(s) + V_z g(s) \right] r_\perp ds \quad (1)$$

$$\phi' = \frac{1}{2G\Omega_1} \oint_{C_1} \frac{q}{t} ds \quad (2)$$

$$\phi' = \frac{1}{2G\Omega_2} \oint_{C_2} \frac{q}{t} ds \quad (3)$$

$$V_y d_y + V_z d_z = 2\Omega_1 q_1 + 2\Omega_2 q_2 + \sum_{m=1}^{2} \oint_{C_m} \left[V_y f(s) + V_z g(s) \right] r_\perp ds \quad (1)$$

$$\phi' = \frac{1}{2G\Omega_1} \oint_{C_1} \frac{q}{t} ds \quad (2)$$

$$\phi' = \frac{1}{2G\Omega_2} \oint_{C_2} \frac{q}{t} ds \quad (3)$$

1. If the shear forces and their locations are known, then q_1 and q_2 are first found in terms of the unknown ϕ' from Eqs. (2) and (3). These q_m 's are then placed into Eq.(1) which is solved for the unknown ϕ' . Once ϕ' is known in this manner, the q_m 's are completely determined.

2. If the shear forces are known but assumed to act through the shear center (whose position is unknown), we can set ϕ' =0, V_y = 0 and solve Eqs. (2) and (3) for the unknowns q_1 and q_2 . Then Eq.(1) gives the location of the shear center, d_z , since

$$V_z d_z = 2\Omega_1 q_1 + 2\Omega_2 q_2 + \sum_{m=1}^{2} \oint_{C_m} \left[V_z g(s) \right] r_\perp ds$$

We can repeat this process by setting $\phi' = 0$, $V_z = 0$ and solving Eqs.(2) and (3) again for new values q_1 and q_2. Then Eq.(1) gives the location of the shear center, d_y :

$$V_y d_y = 2\Omega_1 q_1 + 2\Omega_2 q_2 + \sum_{m=1}^{2} \oint_{C_m} \left[V_y f(s) \right] r_\perp ds$$