tractions, normal stress

```
>> stress = [ 20 40  30;
              40 -10  5;
              30  5   20];
```

stress state

```
>> normal = [3  -4  5]/sqrt(9 +16 +25)

normal =
0.4243   -0.5657    0.7071
```

unit normal to a plane

```
>> norm(normal)

ans = 1

>> tract = normal*stress

tract =
 7.0711   26.1630   24.0416
```

traction vector on plane whose normal is "normal"

```
>> snn = normal*tract'

snn =
   5.2000
```

$$\sigma_{nn} = \mathbf{T}^{(n)} \cdot \mathbf{n}$$

```
>> snn = normal*stress*normal'

snn =
   5.2000
```

$$\sigma_{nn} = \{n\}[\sigma]\{n\}^T$$

total shear stress

```
>> tots = sqrt(norm(tract)^2 -snn^2)

tots =

   35.8533

>> tots_dir = (tract - snn*normal)/tots

tots_dir =

    0.1357    0.8118    0.5680
```

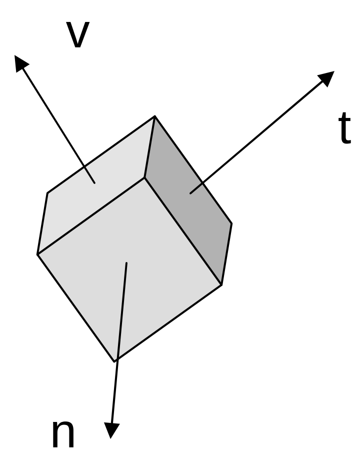
magnitude of total shear stress

direction of total shear stress

```
>> tract*tots_dir'

ans =

   35.8533
```

check: $\tau_s = \mathbf{T}^{(n)} \cdot \mathbf{e}_s$

```
>> n = normal

n =
  0.4243   -0.5657    0.7071

>> t = cross(n, [ 0 0 1])

t = -0.5657   -0.4243         0

>> t = t/norm(t)

t = -0.8000   -0.6000         0

>> v=cross(n,t)

v = 0.4243   -0.5657   -0.7071
```



traction on plane whose normal is t

```
>> tract_on_t = t*stress

tract_on_t =
 -40   -26   -27

>> l = [n' t'  v']

l =
    0.4243   -0.8000    0.4243
   -0.5657   -0.6000   -0.5657
    0.7071         0   -0.7071
```

direction cosines for n, t ,v direction

```
>> stresses_on_t =t*stress*l

stresses_on_t =

  -21.3546   47.6000   16.8291

>> ntv_stresses = l'*stress*l

ntv_stresses =

    5.2000  -21.3546  -28.8000
  -21.3546   47.6000   16.8291
  -28.8000   16.8291  -22.8000
```

stresses on plane whose normal is t

complete stress state on n,t,v planes

principal stresses

```
>> stress

stress =

    20    40    30
    40   -10     5
    30     5    20

>> [pds, pvls] = eig(stress)

pds =

    0.6125   -0.2999    0.7314
   -0.7532   -0.5020    0.4250
   -0.2397    0.8112    0.5334


pvls =

  -40.9344         0         0
         0    5.8148         0
         0         0   65.1196
```

principal directions

state of stress in principal planes

stress invariants

$$\sigma_p^3 - I_1\sigma_p^2 + I_2\sigma_p - I_3 = 0$$

```
function [I1, I2, I3] = stress_invs(s)
I1 = trace(s);
I2 = s(1,1)*s(2,2) - s(1,2)^2 + s(1,1)*s(3,3) - s(1,3)^2 ...
    + s(2,2)*s(3,3) - s(2,3)^2;
I3 = det(s);
```

```
>> [I1,I2,I3]=stress_invs(stress)

I1 =
30

I2 =
 -2525

I3 =
 -15500
```

```
>> [I1,I2,I3]=stress_invs(pvls)

I1 =
30.0000

I2 =
 -2525

I3 =
 -1.5500e+004
```

principal stresses, direction by hand

```
>> stress2 = [ 120  -55  -75;
               -55   55   33;
                -75  33  -85]

stress2 =

   120   -55   -75
   -55    55    33
   -75    33   -85

>> [I1,I2,I3] = stress_invs(stress2)

I1 =
 90

I2 =
 -18014


I3 =
     -471680
```

compute stress invariants

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$I_2 = \sigma_{xx}\sigma_{yy} + \sigma_{xx}\sigma_{zz} + \sigma_{yy}\sigma_{zz} - \sigma_{xy}^2 - \sigma_{xz}^2 - \sigma_{yz}^2$$

$$I_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\sigma_{xy}\sigma_{xz}\sigma_{yz} - \sigma_{xx}\sigma_{zy}^2 - \sigma_{yy}\sigma_{xz}^2 - \sigma_{zz}\sigma_{xy}^2$$

solve cubic

$$\sigma_p^3 - I_1\sigma_p^2 + I_2\sigma_p - I_3 = 0$$

```
>> roots([1, -I1, I2, -I3])

ans =

  176.7995
 -110.8640
   24.0644
```

check from eigenvalue routine

```
>> [pds, pvals] = eig(stress2)

pds =

    0.2872    0.4654    0.8372
   -0.0944    0.8836   -0.4587
    0.9532   -0.0527   -0.2977


pvals =

 -110.8640         0         0
         0   24.0644         0
         0         0  176.7995
```

solve for principal directions

consider $\sigma_p = 176.8$

$$\left(120-176.8\right)n_x - 55n_y - 75n_z = 0$$

$$-55n_x + \left(55-176.8\right)n_y + 33n_z = 0$$

$$-75n_x + 33n_y + (-85-176.8)n_z = 0$$

consider first two eqs

$$-56.8\frac{n_x}{n_z} - 55\frac{n_y}{n_z} = 75$$

$$-55\frac{n_x}{n_z} - 121.8\frac{n_y}{n_z} = -33$$

solve for $\frac{n_x}{n_z}, \frac{n_y}{n_z}$

$$\frac{n_x}{n_z} = -2.81$$

$$\frac{n_y}{n_z} = 1.54$$

From $n_x^2 + n_y^2 + n_z^2 = 1$ $\quad n_z = \frac{\pm 1}{\sqrt{1 + \left(\frac{n_x}{n_z}\right)^2 + \left(\frac{n_y}{n_z}\right)^2}}$

compare to output of eig:

$$n_x = \mp 0.837$$

$$n_y = \pm 0.459$$

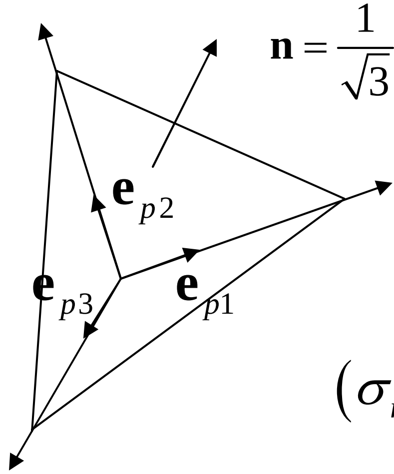
$$n_z = \pm 0.298$$

pds =

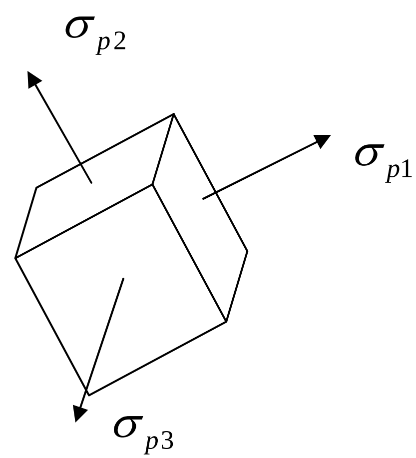
0.2872	0.4654	0.8372
-0.0944	0.8836	-0.4587
0.9532	-0.0527	-0.2977

similarly for other two roots.

stresses on the octahedral plane



$$\mathbf{n}=\frac{1}{\sqrt{3}}\mathbf{e}_{p1}+\frac{1}{\sqrt{3}}\mathbf{e}_{p2}+\frac{1}{\sqrt{3}}\mathbf{e}_{p3}$$



$$\left(\sigma_{nn}\right)_{oct}=\frac{\sigma_{p1}+\sigma_{p2}+\sigma_{p3}}{3}$$

$$\left|\tau_s\right|_{oct}=\frac{1}{3}\sqrt{\left(\sigma_{p1}-\sigma_{p2}\right)^2+\left(\sigma_{p1}-\sigma_{p3}\right)^2+\left(\sigma_{p2}-\sigma_{p2}\right)^2}$$

Effective (von Mises) stress

for $\sigma_{xx}=\sigma$ = the the only non-zero stress $\left|\tau_s\right|_{oct}=\frac{\sqrt{2}}{3}\sigma$

so define the effective stress as:

$$\sigma_{eff}=\frac{3}{\sqrt{2}}\left|\tau_s\right|_{oct}$$

$$=\frac{\sqrt{\left(\sigma_{p1}-\sigma_{p2}\right)^2+\left(\sigma_{p1}-\sigma_{p3}\right)^2+\left(\sigma_{p2}-\sigma_{p2}\right)^2}}{\sqrt{2}}$$

Calculation of octahedral stresses

```
>> stress= [ 10  -3  0;
             -3  -7   2;
               0  2  5]

stress =

    10    -3     0
    -3    -7     2
     0     2     5

>> [pds, pvls] =eig(stress)

pds =

    0.1641    0.0884    0.9825
    0.9746    0.1390   -0.1753
   -0.1521    0.9863   -0.0633


pvls =

   -7.8172         0         0
         0    5.2819         0
         0         0   10.5353
```

```
pvls =

   -7.8172         0         0
         0    5.2819         0
         0         0   10.5353


>> [sig, tau ] = octahedral_stresses(pvls)

sig =
2.6667


tau =
7.7172
```

```
function [ n_oct, tau_oct]= octahedral_stresses(p_stresses)
p= p_stresses;
n_oct = trace(p)/3;
d1 = p(1,1) - p(2,2);
d2 = p(2,2) - p(3,3);
d3 = p(1,1) - p(3,3);
tau_oct = sqrt(d1^2 + d2^2 +d3^2)/3;
```

Note: we can also use our stress transformations directly as long as we remember we know the octahedral plane normal relative to the principal directions explicitly

```
pvls =

   -7.8172         0         0
         0    5.2819         0
         0         0   10.5353
```

principal stress state

```
>> np =[1  1 1]/sqrt(3)

np =
    0.5774    0.5774    0.5774
```

normal of the octahedral plane (relative to principal directions)

```
>> sig = np*pvls*np'

sig =
    2.6667
```

normal stress on the octahedral plane

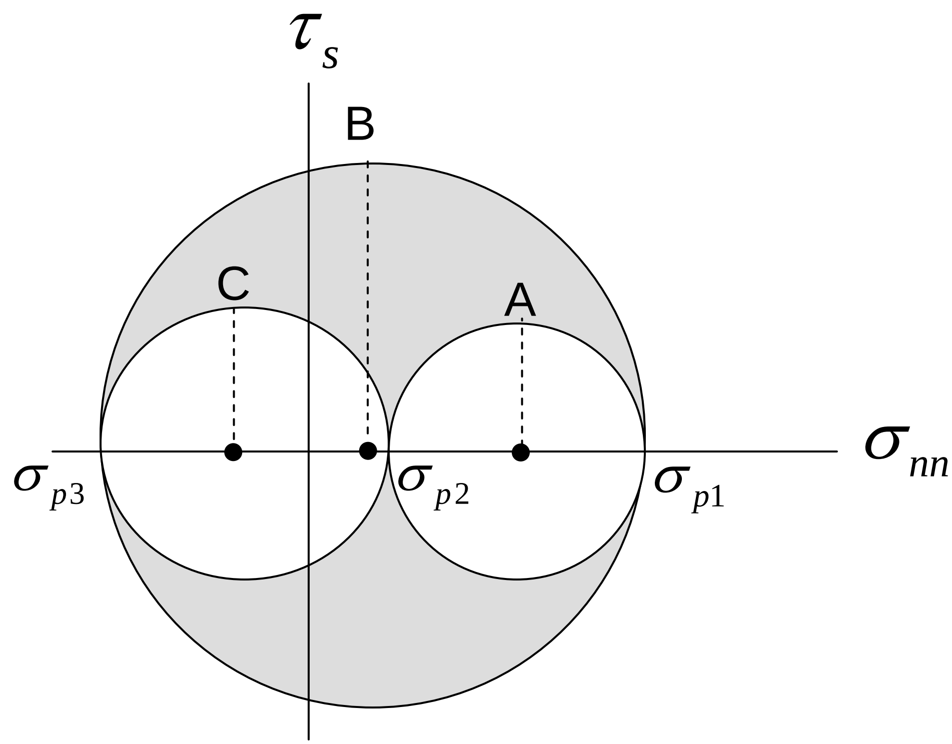
```
>> tract_oct = np*pvls

tract_oct =
   -4.5133    3.0495    6.0826
```

stress vector on the octahedral plane

```
>> tau = sqrt(norm(tract_oct)^2 - sig^2)

tau =
    7.7172
```

total shear stress on the octahedral plane

Mohr's circle(s) for 3-D state of stress



max shear stress

$$|\tau_{\max}| = \max \begin{cases} |\sigma_{p1} - \sigma_{p2}|/2 & \text{A} \\ |\sigma_{p1} - \sigma_{p3}|/2 & \text{B} \\ |\sigma_{p3} - \sigma_{p2}|/2 & \text{C} \end{cases}$$

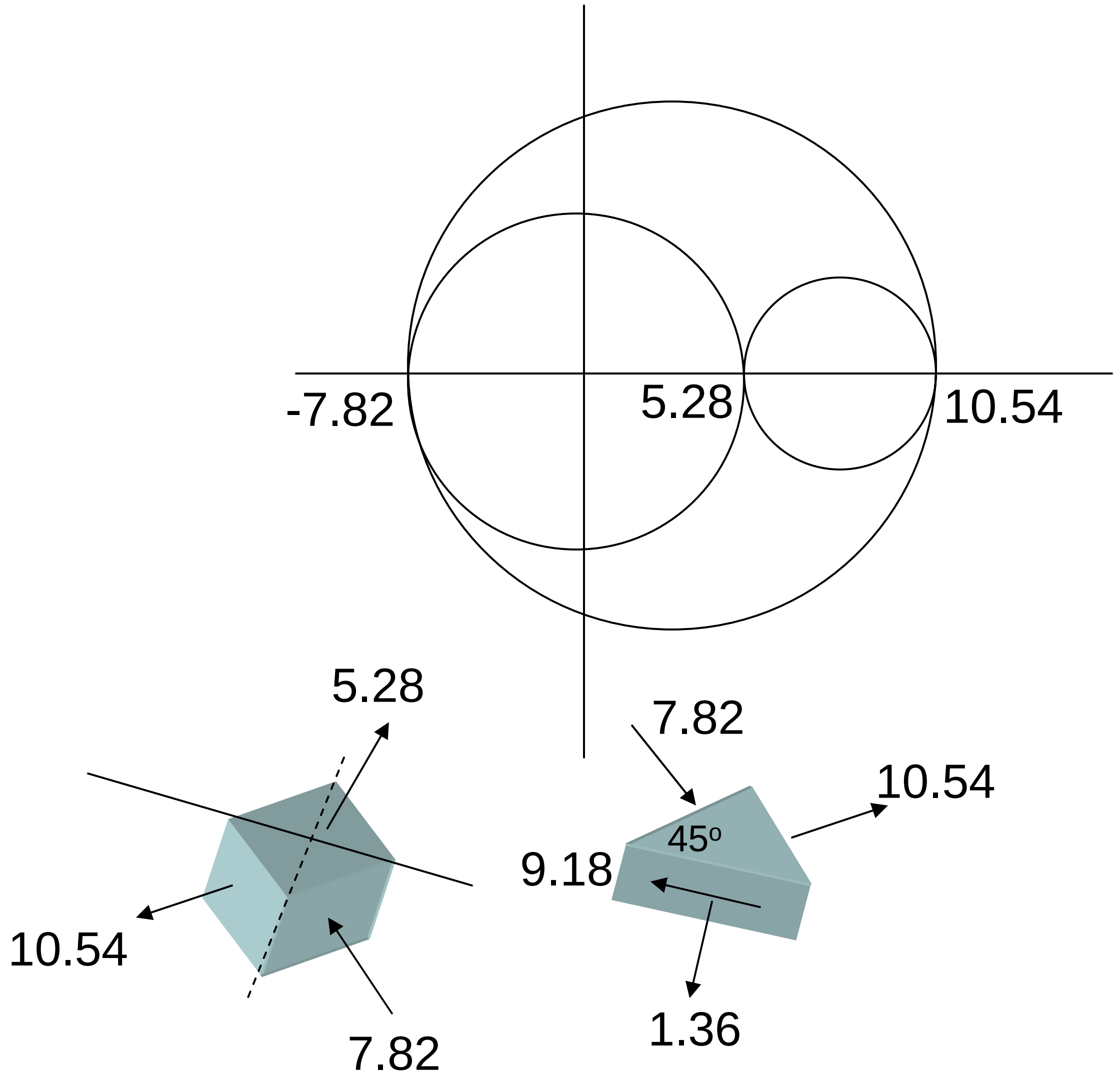
normal stresses on τ_{max} planes

$$\frac{\sigma_{p1} + \sigma_{p2}}{2} \quad \text{A}$$

$$\frac{\sigma_{p1} + \sigma_{p3}}{2} \quad \text{B}$$

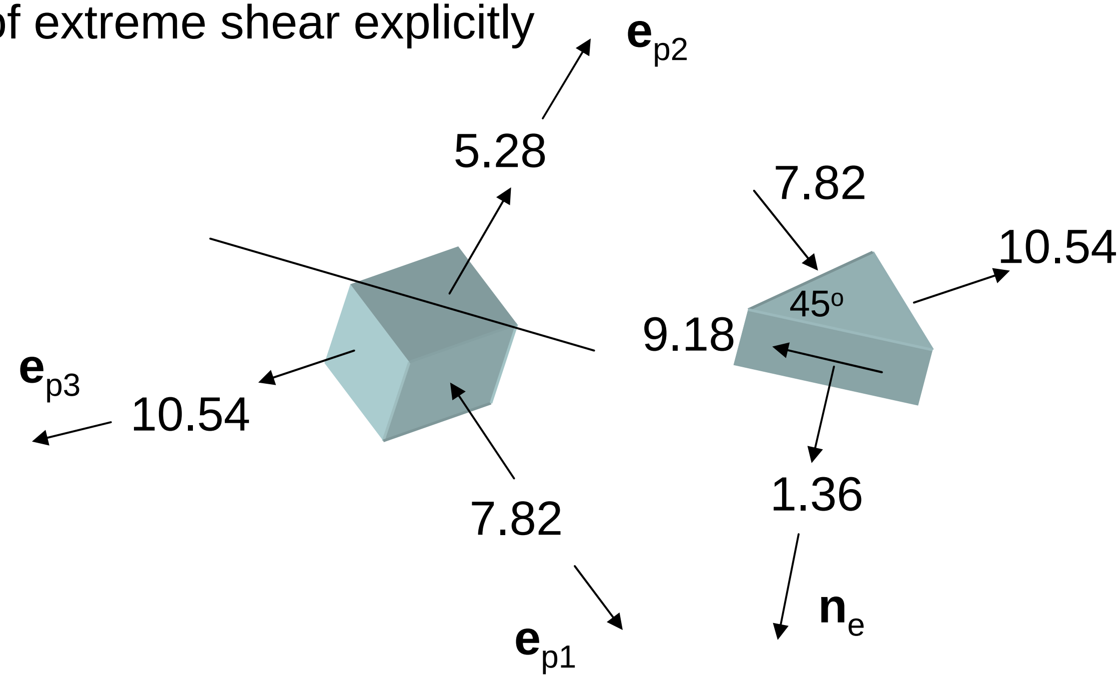
$$\frac{\sigma_{p3} + \sigma_{p2}}{2} \quad \text{C}$$

```
stress =

    10    -3     0
    -3    -7     2
     0     2     5

>> [pds, pvls] = eig(stress)

pds =

    0.1641    0.0884    0.9825
    0.9746    0.1390   -0.1753
   -0.1521    0.9863   -0.0633


pvls =

   -7.8172         0         0
         0    5.2819         0
         0         0   10.5353

>> tau_max = (pvls(3,3) - pvls(1,1))/2

tau_max =

    9.1763

>> norm_stress = (pvls(1,1) + pvls(3,3))/2

norm_stress =
 1.3591
```



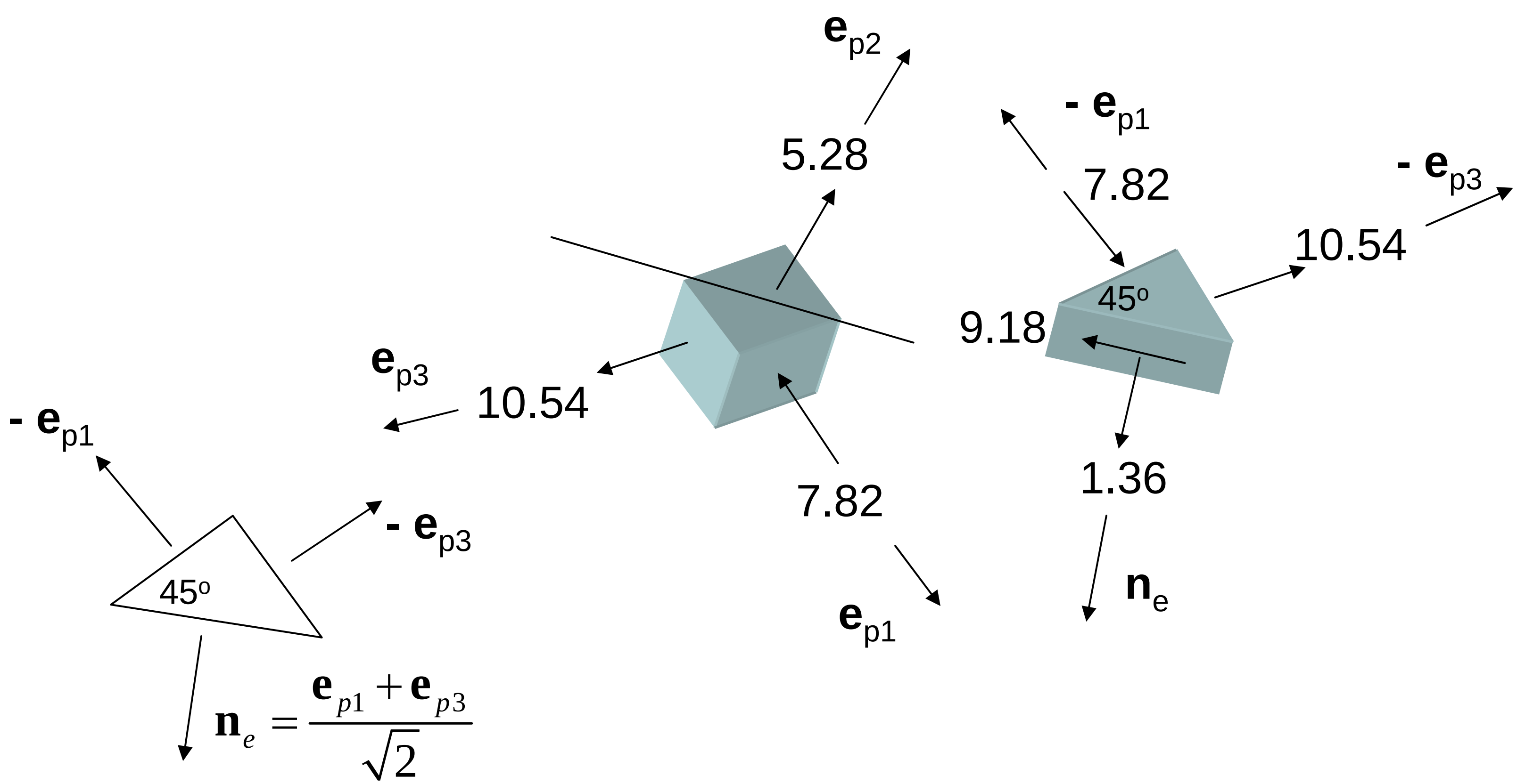
Finding the plane of extreme shear explicitly

```
pds =

   0.1641    0.0884    0.9825
   0.9746    0.1390   -0.1753
  -0.1521    0.9863   -0.0633

>> ep1 =pds(:,1)

ep1 =
    0.1641
    0.9746
   -0.1521

>> ep2=pds(:,2)

ep2 =
    0.0884
    0.1390
    0.9863

>> ep3=pds(:,3)

ep3 =
    0.9825
   -0.1753
   -0.0633
```



principal directions

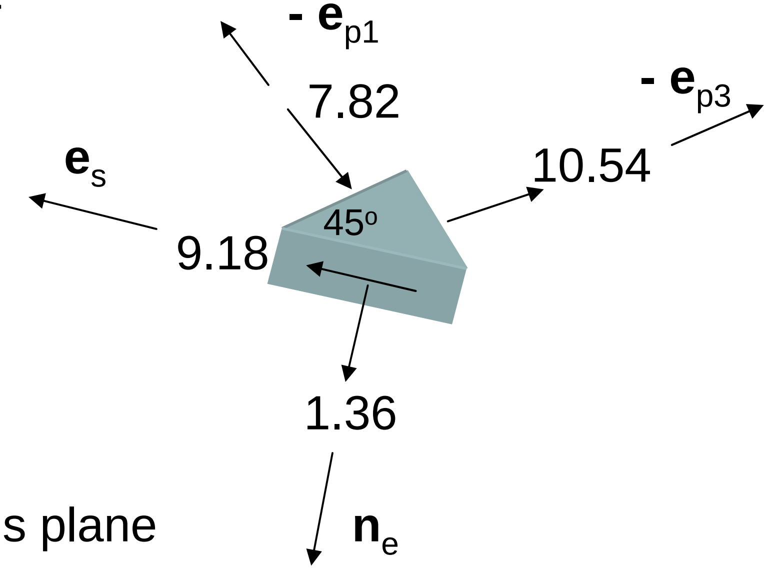
How do we find the unit normal, $\mathbf{n}_e$, for the plane of extreme shear shown ?



$$\mathbf{n}_e = \frac{\mathbf{e}_{p1} + \mathbf{e}_{p3}}{\sqrt{2}}$$

```
>> ne =(ep1+ep3)/sqrt(2)

ne =

    0.8108
    0.5652
   -0.1523
```

Now, we can calculate anything we want about
the stresses on this plane of extreme shear




```
stress =

   10    -3     0
   -3    -7     2
    0     2     5
```

note transpose because n_e is a column vector here

```
>> tract = ne'*stress

tract =
   6.4119   -6.6935    0.3688
```

stress vector on this plane of extreme shear

```
>> norm_stress = ne'*stress*ne

norm_stress =
   1.3591
```

normal stress

```
>> tau_max = sqrt(norm(tract)^2-norm_stress^2)

tau_max =
   9.1763
```

max shear stress (this is total shear on this plane)

```
>> es = (tract - norm_stress*ne')/tau_max

es =
   0.5787   -0.8131    0.0628
```

max shear stress direction