S.0 In MATLAB write a function \( y = \text{box}(t) \); that returns the the "box" function given below as a function of the time, \( t \). [Hint: use logical vectors]

Use the function \( \text{s_space} \) to generate a vector of 512 time values over the interval \([0, 10.24)\) and evaluate the output of the box function with these values and plot the result to verify that the box function is working properly.

Use the MATLAB fast Fourier transform function \( yf = \text{FourierT}(y, dt) \); to determine the complex frequency spectrum of this signal. Set up a set of sampled frequency values using \( f = \text{s_space}(0, 1/dt, 512) \); and plot the magnitude and phase of this spectrum. From the magnitude plot does it appear there is aliasing present here? Do these plots look like the plots shown in Fig. A.2? What is different?

Use the function \( yi = \text{IFourierT}(yf, dt) \); to invert the results back into the time domain. Plot the results versus time. Why do you get a warning message? Verify that you obtain the function box(t), including the values of 0.5 at \( t = 0 \) and \( t = 1 \).

\[
\text{box}(t) = \begin{cases} 
0.5 & t = 0 \\
1.0 & 0 < t < 1 \\
0.5 & t = 1 \\
0 & \text{otherwise}
\end{cases}
\]
S.1 Consider a time domain signal given by

\[ y(t) = \begin{cases} \cos(3\pi t) & -0.5 < t < 0.5 \\ 0 & \text{otherwise} \end{cases} \]

where \( t \) is measured in \( \mu \text{sec} \)

1. Evaluate and plot this signal in MATLAB from \( t = -4 \ \mu\text{sec} \) to \( t = +4 \ \mu\text{sec} \). Obtain the Fourier transform of this function analytically (Hint: the function is even in \( t \) so what does this imply about its Fourier transform?) and plot it in MATLAB from zero to 14MHz.

(a) What is the value of the Fourier transform at zero frequency? How is this value related to the properties of \( y(t) \)?

(b) At what frequency is the Fourier transform a maximum? How is this frequency related to the properties of \( y(t) \)?

2. From the samples of \( y(t) \) over the time interval \((-4, 4)\) obtain the Fourier transform numerically with the MATLAB FFT function FourierT. Plot the magnitude of this function and show that it agrees with the magnitude of your analytical result. Use IFourierT and show that you recover the \( y(t) \) function.
S.2 When a P-wave strikes a stress-free planar interface of an elastic solid, both reflected P-waves and SV-waves are generated:

![Diagram showing incident and reflected waves](image)

The MATLAB function `stress_freeP` can be used to obtain the reflection coefficients (based on velocity ratios) for these reflected waves. The polarizations of these waves (i.e. the assumed directions of the velocity) are shown by the black arrows.

(a) plot these reflection coefficients versus the incident angle for angles from 0 to 90 degrees). Justify the values you obtain for these reflection coefficients when the incident angle is zero.
The mode conversion present in this free surface interaction can be used to generate an SV-wave traveling normal to the surface of a component that is being inspected by using a P-wave transducer and a wedge as shown:

(b) Determine the wedge angle, $\theta$, that is needed to generate a normally incident SV-wave if the wedge is made of aluminum.

(c) Determine the amplitude, $A$, of velocity of the SV-wave transmitted into the component being inspected if the component is made of steel. Assume the incident P-wave amplitude is unity and assume perfect contact between the aluminum wedge and the steel. Use plane wave relations to determine $A$ even though the transducer does not generate purely plane waves.
S.3. A plane wave travels 100 mm in a material to a point where its amplitude is $P_1$. After the wave travels through an additional 100 mm of material its amplitude is reduced to $P_2 = 0.35 P_1$. What is the average attenuation of this material in dB/m?

S.4. A transducer beam spreads as it propagates. In the far-field of the transducer this spreading is just like that of a spherical wave, i.e. it varies as $1/r$, where $r$ is the distance from the transducer. At a distance of 100 mm from the transducer the amplitude of the pressure is $P_1$. After the beam has propagated an additional 100 mm the amplitude is reduced to $P_2 = 0.35 P_1$. What is the average attenuation of the material in dB/m?
S.5. Consider a harmonic plane P-wave traveling in water at room temperature. Determine an expression for the distance (as a function of frequency) that this wave must travel to reduce its amplitude by 10% due to attenuation. Plot this function for $f = 1$ MHz to $f = 20$ MHz.