Virtual Work-Finite Element Problems

1. Consider the three springs shown below. There are 4 nodal displacements at the ends of these springs so the global stiffness matrix, \( K \), is 4x4 and the force vector, \( F \), corresponding to those nodes is a 4x1 vector. Starting with zero values for \( K \) and \( F \), show (using the given numbering scheme):

(a) the new values for \( K \) and \( F \) when we add in the contributions from spring 1,
(b) the new values for \( K \) and \( F \) when we add in the contributions from spring 2,
(c) the new values for \( K \) and \( F \) when we add in the contributions from spring 3.
(d) Show the modified values of \( K \) and \( F \) after we apply the boundary conditions on force and displacement.
(e) Solve for the nodal displacements
(f) Obtain the unknown reaction force and the forces (T or C) in each spring from those displacements

\( k_1 = 20 \text{ lb/in} \)
\( k_2 = 50 \text{ lb/in} \)
\( k_3 = 10 \text{ lb/in} \)
2. Consider the system of hanging weights and springs shown. If the weights \( W = 15 \text{ lb} \), and the spring constants \( k = 20 \text{ lb/in} \), determine:

(a) the nodal displacements (use the given numbering scheme)
(b) the unknown reaction force at the support
(c) the forces (T or C) in each spring
3. Consider the truss shown. The two cross members are not connected to each other. Determine:

(a) the displacements at each node (use the given numbering scheme) in m
(b) the external reactions on the truss in kN
(c) the force in each bar in kN (T or C)

The Young's modulus for each bar is \( E = 70 \text{ GPa} \)
and the cross-sectional area \( A = 0.004 \text{ m}^2 \)