Continuous-time Hammerstein and Wiener modeling under second-order static nonlinearity for periodic process signals

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Abstract

This article presents continuous-time (CT) analytical solutions to Hammerstein and Wiener systems with second-order plus-lead (SOPL) dynamic behavior for sinusoidal input changes. The proposed solutions depend only on the most recent input change and exact accuracy is demonstrated for cases of varying frequency (ω), amplitude (A), and phase angle (ϕ). This article demonstrates two critical advancements in the application of these solutions using a multiple input, multiple output (MIMO) mathematically simulated continuous stirred tank reactor (CSTR). The first one is improved accuracy over approximating periodic input changes as piece-wise step changes. The second one is the ability to accurately model process noise in the outputs when the input process noise can be decomposed into a sum of sinusoidal components. Since in many applications inputs are measured at a much higher rate than an output, the CT modeling of periodic process noise provides a means to model periodic output noise despite infrequent sampling of the output. Moreover, this article also presents output correction using measured output to remove prediction bias under white and serially correlated noise of measured outputs.

Keywords: Nonlinear systems; Continuous-time modeling; Sinusoidal input sequence; Hammerstein and Wiener models

1. Introduction

Periodic phenomena, either forced or natural, exist in many engineering applications. For example, the natural cycle of the upstream processes or environmental influences can cause periodical oscillations. Periodic process behavior also results from periodic control system behavior (e.g., on/off valve action, underdamped control, excessive controller gain), electrical heating systems, and tank level, temperature and pressure fluctuations, to name only a few. Sometimes, forced periodic operations are introduced to a system to improve selectivity and yield. However, periodically time-varying (PTV) systems are non-stationary, more difficult to control, and inherently more complex (Pan & Lee, 2003). Analysis, control, and the system identification of PTV systems are of particular interest in systems engineering. Systems with sinusoidal input behavior, the focus of this article, are a special class of PTV research.

When inputs vary sinusoidally, corresponding outputs will likely vary periodically. Since sinusoidal behavior is continuous-time (CT) behavior, the accurate prediction of this behavior requires fast sampling of the inputs, CT modeling or both. More specifically, a discrete-time (DT) model will only be able to predict output periodic behavior under fast sampling and although a CT model based on step input changes can provide prediction continuously over time, its accuracy will suffer when sampling is too slow to adequately approximate periodic input behavior (a phenomenon called aliasing, see Seborg, Edgar, & Mellichamp, 2003). The only way to overcome these limitations is to exactly model the CT periodic behavior of the input signal, which can only be accomplished with a CT model. Thus, in this article, we present a continuous-time, non-linear,
dynamic approach that exactly models periodic input signals. More specifically, the scope of our work is the treatment of sinusoidal inputs that are imposed on sequential step changes in processes that can be accurately modeled by Hammerstein and Wiener systems (see Gomez & Baeyenes, 2004; Hagenblad & Ljung, 2000; Park, Sung, & Lee, 2006; Zhu, 2002 for recent work for these systems) with second-order plus-lead (SOPL) dynamic behavior. Since this article addresses prediction only, all model coefficients are assumed known and process identification is outside the scope of this article. This research provides exact solutions under the scope of this work that depend only on the most recent input change, a property that we call “compact.”

In Section 2 we briefly describe the Hammerstein and Wiener systems and give our proposed CT solutions under the scope of this work. We also demonstrate their perfect accuracy for true Hammerstein and Wiener systems in cases of varying frequency (ω), amplitude (A) and phase angle (φ). In Section 3 we present two studies using a mathematically simulated CSTR. The first compares the CT “W-BEST” method of Bhandari and Rollins (2003) that uses approximate piece-wise step change sampling and our proposed approach using the true behavior of the sinusoidal inputs. Although, the inputs are sampled frequently, this study will demonstrate critical advantages of the proposed method when process inputs are periodic. In the second study, we add large amounts of sinusoidal process noise to the inputs to demonstrate the advantages of the proposed method over DT and CT piece-wise step input methods. In this study we also demonstrate the use of measured outputs to correct prediction bias under sampling of outputs. Thus, we demonstrate the incorporation of measured outputs under white noise and seriously correlated noise as it combines with predicted process noise to correct prediction bias.

2. The Hammerstein and Wiener solutions (algorithms)

Hammerstein and Wiener models belong to the class of block-oriented models, which are series or parallel combinations of linear dynamic blocks and static nonlinear mappings. A Hammerstein model consists of a static nonlinear mapping or gain followed by a linear dynamic block. The input vector \( u(t) \) goes through the nonlinear static mapping block and gives an unobservable intermediate vector \( v(t) \). Then \( v(t) \) goes through the linear dynamic block to give the output vector, \( y(t) \), with \( f(u(t)) \) and \( g(t) \) denoting the nonlinear static gain function vector and the linear dynamic function vector, respectively; with \( v(t) = f(u(t)) \) and each element of \( y(t) \) is \( y_i = \int_0^t v_i(ξ)g_i(t-ξ)dξ \). The Wiener system has the same two type of blocks as the Hammerstein system, but in the reverse order. In the Wiener structure, the input vector \( u(t) \) is transformed by the linear dynamic block to get \( v(t) \), and then \( v(t) \) goes through the static nonlinear block to produce \( y(t) \). Mathematically, each element of \( v(t) \) is

\[
v_i = \int_0^t u_i(ξ)g_i(t-ξ)dξ \quad \text{and} \quad y(t) = f(v(t)).
\]

The Hammerstein and Wiener systems have simple structures and are among the most popular block-oriented nonlinear models. They can accurately represent many processes with nonlinear static and dynamic characteristics and have been employed to model complex chemical processes, including pH neutralization (Zhu & Seborg, 1994; Kalafatis, Arifin, Wang, & Cluett, 1995; Norquay, Palazoglu, & Romagnoli, 1999a,b; Eskinat, Johnson, & Luyben, 1991) and polymerization process (Su & McAvo, 1993). Due to their simple structures and parametric nature, the Hammerstein and Wiener modeling applications are numerous and growing rapidly. However, most modeling problem have involved the use of discrete-time methods (Doyle, Pearson, & Oggunna, 2002; Greblicki, 1992, 1997; Wigren, 1993); exception include Greblicki (1999), Greblicki (2000), Rollins, Bhandari, Bassily, Colver, and Chin (2003), and Bhandari and Rollins (2003) who employed CT approaches. The method proposed by Greblicki (2000) was nonparametric and the dynamic block was identified using an impulse response approach. Rollins and coworkers introduced a parametric approach they called H- or W-BEST for their Hammerstein and Wiener approaches, respectively. An important drawback of these methods is that they approximate all inputs in the same way as discrete-time methods, as piece-wise step input sequences. This section introduces extensions of H- and W-
BEST to directly treat sinusoidal inputs imposed on step changes that give exact solutions to Hammerstein and Wiener systems with SOPL dynamics. For algorithms for first order and second order overdamped systems see Zhai, Rollins, and Bhandari (2006). In this article, all variables are considered to be deviation variables from an initial steady state.

The general form of the input that we address in this work is:

\[
\mathbf{u}(t) = \begin{cases} 
0 & t \leq 0 \\
 b_1 + A_1 \sin(\omega_1 t + \phi_1) & 0 < t \leq t_1 \\
 b_2 + A_2 \sin(\omega_2(t - t_1) + \phi_2) & t_1 < t \leq t_2 \\
 \vdots & \vdots \\
b_n + A_n \sin(\omega_n(t - t_{n-1}) + \phi_n) & t_{n-1} < t \leq t_n
\end{cases}
\]  

(1)

As one sees from Eq. (1), for each time interval, the deviation of the input from the initial steady state is composed of a different step component and a sinusoidal component with different amplitude, frequency, and phase. Next, in Section 2.1, we use Eq. (1) as an input to the Hammerstein system with SOPL dynamics and present our proposed CT solution and demonstrate its exact accuracy under varying \(A_i\), \(\omega_i\), and \(\phi_i\). In Section 2.2 we present these results for the Wiener system.

2.1. The Hammerstein solution

The SOPL linear dynamic block for the Hammerstein system can be written as:

\[
\tau_1 \frac{d^2 y(t)}{dt^2} + (\tau_1 + \tau_2) \frac{dy(t)}{dt} + y(t) = \tau_a \frac{dv(t)}{dt} + v(t)
\]  

(2)

with the statistic nonlinear function written in general form as

\[
v(t) = \mathbf{f}(\mathbf{u}(t))
\]  

(3)

Note that no restrictions are placed on the function \(\mathbf{f}(\mathbf{u}(t))\). However, for convenience, we will present our solution for the quadratic form of Eq. (3) given below.

\[
\mathbf{f}(\mathbf{u}(t)) = a_1 \mathbf{u}(t) + a_2 \mathbf{u}^2(t)
\]  

(4)

Using Eq. (1) as an input into this system of equations and the form of \(\mathbf{f}(\mathbf{u}(t))\) given by Eq. (4), we propose Eq. (5) below as a solution:

\[
y(t) = (a_1 b_1 + a_2 b_1^2 + a_2 A_1^2/2)g_20(t - t_{i-1}; \tau_1, \tau_2) + (a_1 A_1 + 2a_2 b_1 A_1)(\cos \phi_1 - \omega_1 \tau_a \sin \phi_1)g_{2s}
\times (t - t_{i-1}; \omega_1, \tau_1, \tau_2) + (a_1 A_1 + 2a_2 b_1 A_1)
\times (\sin \phi_1 + \omega_1 \tau_a \cos \phi_1)g_{2s}(t - t_{i-1}; \omega_1, \tau_1, \tau_2)
\times (t - t_{i-1}; 2\omega_1, \tau_1, \tau_2) + a_2 A_1^2/2(2\sin \phi_1 + 2\omega_1 \tau_a \cos \phi_1)g_{2s}
\times (t - t_{i-1}; 2\omega_1, \tau_1, \tau_2) + 2(2\omega_1 \tau_a \sin 2\phi_1 - \cos 2\phi_1)g_{2s}(t - t_{i-1}; 2\omega_1, \tau_1, \tau_2)
+ y(t_{i-1})g_{12}(t - t_{i-1}; \tau_1, \tau_2) + y'(t_{i-1})g_{12}
\times (t - t_{i-1}; \tau_1, \tau_2)
\]  

(5)

where \(t_i\) is the time of the \(i\)th sampling instant, and the \(g_i\)’s are defined below.

\[
g_{20}(t; \tau_1, \tau_2) = 1 + \frac{\tau_1}{\tau_2 - \tau_1} e^{-t/\tau_1} - \frac{\tau_2}{\tau_2 - \tau_1} e^{-t/\tau_2}
\]  

(6)

\[
g_{2s}(t; \omega, \tau_1, \tau_2)
\]  

\[
= \frac{\omega \tau_2^2 e^{-t/\tau_1}}{(\tau_1 - \tau_2)(1 + \omega^2 \tau_1^2)} + \frac{\omega \tau_1^2 e^{-t/\tau_2}}{(\tau_2 - \tau_1)(1 + \omega^2 \tau_2^2)}
\]  

(7)

\[
g_{2c}(t; \omega, \tau_1, \tau_2)
\]  

\[
= -\frac{\tau_1 e^{-t/\tau_1}}{(\tau_1 - \tau_2)(1 + \omega^2 \tau_1^2)} - \frac{\tau_2 e^{-t/\tau_2}}{(\tau_2 - \tau_1)(1 + \omega^2 \tau_2^2)}
\]  

(8)

\[
g_{02}(t; \tau_1, \tau_2) = \frac{\tau_1 e^{-t/\tau_1} - \tau_2 e^{-t/\tau_2}}{\tau_1 - \tau_2}
\]  

(9)

\[
g_{12}(t; \tau_1, \tau_2) = \frac{\tau_1 \tau_2 e^{-t/\tau_1} - \tau_1 \tau_2 e^{-t/\tau_2}}{\tau_1 - \tau_2}
\]  

(10)

In Fig. 1 we evaluate the proposed solution for a specific case of coefficients (given in the figure) where we varied \(A_i\), \(b_i\), and \(\omega_i\) as shown in the figure. As shown by the output response \(y\),

![Fig. 1. Sinusoidal input (u) on the left, and true and predicted output by Eq. (5) on the right. The process is a true Hammerstein system with \(\phi_i = 0\), \(a_1 = 1.0\), \(a_2 = 2.0\), \(\tau_1 = 5.0\), \(\tau_2 = 3.0\), \(\tau_a = 2.0\), \(\omega_1\); varying from 0.4 to 3.0, and \(b_i\) and \(A_i\) values varying arbitrarily.](image-url)
the figure to the right, Eq. (5) predictions agree exactly with the true Hammerstein system. In Fig. 2 we varied \( A_i, \omega_i, \phi_i \) but not the step changes. Again, exact agreement is obtained by the proposed approach with the true process. Note that for multiple inputs, Eq. (4) is written to include all the inputs (see Rollins et al., 2003).

2.2. The Wiener solution

The SOPL linear dynamic block for the Wiener system can be written as:

\[
\tau_1 \tau_2 \frac{d^2 v(t)}{dt^2} + (\tau_1 + \tau_2) \frac{dv(t)}{dt} + v(t) = -\frac{du(t)}{dt} + u(t)
\]

(11)

with the static nonlinear block given as

\[
y(t) = f(v(t))
\]

(12)

Using Eq. (1) as an input into Eq. (11), we propose the following CT solution for \( v(t) \):

\[
v(t) = b_i g_{20}(t - t_{i-1}; \tau_1, \tau_2) + A_i (\cos \phi_i - \omega_i \tau_a \sin \phi_i) g_{2s} \\
\times (t - t_{i-1}; \omega_i, \tau_1, \tau_2) + A_i (\sin \phi_i + \omega_i \tau_a \cos \phi_i) g_{2c} \\
\times (t - t_{i-1}; \omega_i, \tau_1, \tau_2) + v(t_{i-1}) g_{02} (t - t_{i-1}; \tau_1, \tau_2) \\
+ v'(t_{i-1}) g_{12} (t - t_{i-1}; \tau_1, \tau_2) \quad \text{for} \quad t_{i-1} < t \leq t_i
\]

(13)

Thus, Eq. (13) with Eq. (12) gives the proposed solution for the Wiener system. Note that this solution is given in terms of a general form for Eq. (12). In Fig. 3, we evaluate the proposed Wiener solution for a specific case of coefficients (given in the figure) where we varied \( A_i, b_i, \) and \( \phi_i \) as shown in the figure. For Eq. (12), we used the quadratic form given below:

\[
f(v(t)) = a_1 v(t) + a_2 v^2(t)
\]

(14)

In Fig. 3, as shown by the output response, the proposed solution agrees exactly with the true Wiener system. In Fig. 4, \( \phi_i \) is allowed to vary. As in the other cases, agreement between the proposed solution and the true process is exact. By comparing the output responses in Fig. 1 with Fig. 3 and Fig. 2 with Fig. 4, one can contrast the Hammerstein response with the Wiener response, although the coefficients are the same for both systems. Note that for multiple inputs, Eq. (12) is written to include all the inputs (see Bhandari & Rollins, 2003).

3. Cases using a mathematically simulated CSTR

In this section we evaluate the proposed approach using the Wiener model Bhandari and Rollins (2003) developed for the continuous stirred-tank reactor (CSTR) appearing in this work and shown in Fig. 5 below. The second-order, exothermic reaction taking place in the CSTR gives the process strong nonlinear and interactive behavior.
Fig. 4. Sinusoidal input (u) on the left, and true and predicted output by Eqs. (13) and (14) on the right. The process is a true Wiener system described above with 

\[ b_i = 0, \]

where \( f \) is a quadratic polynomial function with \( a_1 = 1.0, a_2 = 2.0, \tau_1 = 5.0, \tau_2 = 3.0, \tau_3 = 2.0, \omega_i \) varying from 0.4 to 1.5, and \( \phi_i \) and \( A_i \) values varying arbitrarily.

The process model consists of the overall mass balance, component (A and B) mole balances, and energy balances on tank and jacket contents. The input variables are the feed flowrate of A \( (Q_A) \), the feed temperature of A \( (T_{AF}) \), the feed concentration of A \( (C_{Af}) \), the feed flowrate of B \( (Q_B) \), the feed temperature of B \( (T_{Bf}) \), the feed concentration of B \( (C_{Bf}) \) and the coolant flowrate to the jacket \( (Q_c) \). The output variables are the concentrations of species A, B and C in the reactor (i.e., \( C_A, C_B, \) and \( C_C \), respectively), the temperature in the reactor \( (T_t) \) and the coolant temperature \( (T_c) \) in the jacket. Thus, this process consists of seven (7) inputs and five (5) outputs. For more details of this process see Bhandari and Rollins (2003). Bhandari and Rollins (2003) built accurate models for all the outputs using their CT Wiener method (i.e., W-BEST) with second order non-linear static functions and SOPL dynamic functions. In our study, we use this Wiener model of the CSTR to highlight the advantages of the proposed method. More specifically, we impose sinusoidal and step changes on the inputs to this process and compare the performance of the proposed approach that treats true periodic nature of the input with the CT approach of Bhandari and Rollins (2003) that approximates the inputs as sequential piece-wise step changes. Since the Wiener model is the same for both methods (i.e., the model developed by Bhandari & Rollins, 2003), with
only the treatment of the inputs differing, this study directly addresses the ability of the proposed method to provide better accuracy when periodicity exist in input variables either as signal (Section 3.1) or process noise (Section 3.2). To obtain the static and dynamic forms this Wiener model of the CSTR uses see Bhandari and Rollins (2003).

3.1. Sinusoidal input signal

In this study, the input changes shown in Fig. 6, are introduced into the CSTR. These sinusoidal changes are imposed on step changes of the form \( b_i + A_i \sin(\omega_i(t - t_{i-1})) \). Note that \( b_i, \omega_i, \) and \( A_i \) vary considerably in level between inputs and within inputs (Figs. 7 and 8).

One criterion that we used to evaluate the approaches in this study is the sum of squared prediction error (SSPE) and it was determined for each method by using true and predicted responses of the methods at 300 equally spaced intervals of the response time of 300 min (i.e., at each minute) in the following formula:

\[
SSPE = \sum_{i=1}^{300} (y_i - \hat{y}_i)^2
\]  

where \( y_i \) is the true response and \( \hat{y}_i \) is the predicted response for the specified method.

The second evaluation criterion we used was the average relative error (ARE), which used the same values as SSPE and its formula is given below:

\[
ARE = \frac{1}{300} \sum_{i=1}^{300} \left| \frac{y_i - \hat{y}_i}{y_i} \right|^2
\]  

For this study the SSPE and ARE results are contained in Table 1. Results calculated directly from Eqs. (15) and (16) are given as well as the relative results to PM. Since all the rela-

![Fig. 7. The concentration responses of species A, B, and C as a function of time for the input changes given in Fig. 6. “TRUE” is the true process response, “W-BEST” is the predicted response using the piece-wise step input change method of Bhandari and Rollins (2003) and “PM” is calculated from the proposed method described in Section 2.](image)

![Fig. 8. The temperature responses for the tank contents and the jacket contents as a function of time for the input changes given in Fig. 6. “TRUE” is the true process response, “W-BEST” is the predicted response using the piece-wise step input change method of Bhandari and Rollins (2003) and “PM” is calculated from the proposed method described in Section 2.](image)
The comparison of predictions based on the proposed method (PM) and W-BEST

<table>
<thead>
<tr>
<th>Output</th>
<th>Absolute SSPE values</th>
<th>Relative SSPE values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PM</td>
<td>W-BEST</td>
</tr>
</tbody>
</table>
| (A) SSPE comparison
| $C_A$  | 0.1051                | 0.8373               | 1.000     | 7.964    |
| $C_B$  | 0.1107                | 0.1957               | 1.000     | 1.767    |
| $C_C$  | 0.09910               | 0.3310               | 1.000     | 3.340    |
| $T_t$  | 903.460               | 6368.665             | 1.000     | 7.049    |
| $T_c$  | 209.294               | 1335.62              | 1.000     | 6.382    |

<table>
<thead>
<tr>
<th>Output</th>
<th>Absolute ARE values</th>
<th>Relative ARE values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PM</td>
<td>W-BEST</td>
</tr>
</tbody>
</table>
| (B) ARE comparison
| $C_A$  | 0.00988             | 0.01597             | 1.000     | 1.168    |
| $C_B$  | 0.0267              | 0.03119             | 1.000     | 1.167    |
| $C_C$  | 0.00763             | 0.01073             | 1.000     | 1.407    |
| $T_t$  | 0.000871            | 0.001558            | 1.000     | 1.788    |
| $T_c$  | 0.000478            | 0.000803            | 1.000     | 1.680    |

Note that from the relative SSPE results W-BEST is as much as eight times worse and as much as 80% worse by relative ARE results. Thus, we conclude that our proposed method can offer an improvement in predictive performance when inputs vary sinusoidally. Next we examine performance of PM under composite sinusoidal process noise.

### 3.2. Composite sinusoidal process noise

An input signal can be decomposed into a sum of sinusoidal components with uncorrelated random coefficients with frequencies $\omega \in [0, \pi]$ as any stationary time series according to

$$X(t) = \sum_{j=1}^{k} (A_j \cos(\omega_j t) + B_j \sin(\omega_j t)),$$

where the $A_j$’s and $B_j$’s are uncorrelated random variables with $E[A_j] = 0$, and $\text{Var}(A_j) = \text{Var}(B_j) = \sigma^2$, $j = 1, \ldots, k$. In general, infinitely many sinusoidal terms rather than a finite number should be included in the above equation. However, once the ones with large amplitudes and major frequencies are included, this approximation works reasonable well for a stationary time series (Brockwell & Davis, 2002). Therefore, the sum of sinusoidal components can be employed to approximate noisy step input changes after the spectral decomposition. To obtain periodic components from noisy signals in practice see Hajjari and Eloutassi (1999). In this section we are assuming the decomposition and identification are accurately done on the input variables and we are evaluating the ability of the proposed method to model the noise in the outputs.

The noise that we added to each input for this study is shown in Figs. 9 and 10. Each input is composed of eight sinusoidal functions with different uncorrelated frequencies and ampli-

![Fig. 9. Flow rate inputs showing the added noise levels in deviation variable form.](image-url)
Fig. 10. Composition and temperature inputs showing the added noise levels.

tudes. As one can see, the noise to signal ratio is quite substantial for some of the inputs (e.g., \( C_{Af} \)). For a better view of the noise, we have blown up views included in these figures.

CSTR outputs for these input sequences are shown in Figs. 11 and 12 for \( C_A \). The other output responses are not given for space consideration. The true output response for \( C_A \) is given in all these figures. For the three time intervals 0–100, 100–200, and 200–300 min, these figures also give \( r_{fit} \), the correlation coefficient between the predicted values and the true values. Fig. 11 gives the responses of PM and WB when the input sampling rate, \( \Delta t_i \), is 0.2 min. As this figure shows, the PM give excellent fit to the true response (\( r_{fit} \) is greater than or equal to 0.96 is all three intervals) while the \( r_{fit} \) for WB varies from 0.79 to 0.86. This performance is also seen visually by the blown up views. Fig. 12 gives the WB plots for \( \Delta t_i = 1.0 \) min. As shown, for this case WB performance is very poor with \( r_{fit} \) varying from 0.29 to 0.37. Table 2 shows the drop in \( r_{fit} \) for WB as \( \Delta t_i \) increases. As this table shows, the performance of WB drops considerably as \( \Delta t_i \) goes from 0.2 to 1.0 min. Thus, one sees that the PM

<table>
<thead>
<tr>
<th>Time interval (min)</th>
<th>0–100</th>
<th>100–200</th>
<th>200–300</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM</td>
<td>0.97</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td>WB ( \Delta t_i = 0.2 )</td>
<td>0.86</td>
<td>0.82</td>
<td>0.79</td>
</tr>
<tr>
<td>WB ( \Delta t_i = 0.6 )</td>
<td>0.59</td>
<td>0.66</td>
<td>0.59</td>
</tr>
<tr>
<td>WB ( \Delta t_i = 1.0 )</td>
<td>0.36</td>
<td>0.29</td>
<td>0.37</td>
</tr>
</tbody>
</table>

\( \Delta t_i \) = the input sampling rate in minutes.
correlates well with the output noise but that the input sampling rate can affect WB accuracy significantly depending on sampling rate which determines the accuracy of the piece-wise step approximation.

In Figs. 11 and 12 one sees bias prediction especially in the interval from 200 to 300 min. Fig. 13 illustrates the ability of measured outputs to reduce this prediction bias at an output sampling rate, $\Delta t_0$, equal to 5.0 min. This correction was obtained by modeling the output noise as an autoregressive, integrated, moving average (ARIMA) process (see Box & Jenkins, 1976) following the procedure in Rollins, Bhandari, Chin, Junge, and Roosa (2006). It was found to follow an autoregressive (AR) process with lag one. Thus, the predicted output response in Fig. 13 was obtained from the following algorithm:

$$
\hat{y}(0) = y(0) \\
0 \leq t < \Delta t_0, \quad \hat{y}(t) = \hat{\eta}(t) \\
\Delta t_0 \leq t < 2\Delta t_0, \quad \hat{y}(t) = \hat{\eta}(t) + \hat{\varphi}(y(0) - \hat{\eta}(0)) \\
2\Delta t_0 \leq t < 3\Delta t_0, \quad \hat{y}(t) = \hat{\eta}(t) + \hat{\varphi}(y(\Delta t_0) - \hat{\eta}(\Delta t_0)) \\
\vdots \\
(k-1)\Delta t_0 \leq t < k\Delta t_0, \quad \hat{y}(t) = \hat{\eta}(t) + \hat{\varphi}(y((k-2)\Delta t_0) - \hat{\eta}(k-2)\Delta t_0)
$$

(18)
where $\bar{\phi}$ was estimated as 0.90. As this figure shows, the correction worked well and the fit is excellent. This improvement is especially seen from the blown up view for the interval from 250 to 300 min.

Fig. 14 is the fit of the DT WB method developed by Rollins and Bhandari (2004). This method was compared to the CT WB method and found to perform equally as well when the conditions were appropriate for DT modeling. However, the conditions in Fig. 14 consists of input and output rates of 5.0 min which must be the same when building a DT model. For this case, the output noise was uncorrelated. Under uncorrelated output noise, the predicted response was determined from

$$
\bar{y}(0) = y(0)
$$

$$
0 \leq t < \Delta t_0,
\bar{y}(t) = \bar{\eta}(t)
$$

$$
\Delta t_0 \leq t < 2\Delta t_0,
\bar{y}(t) = \bar{\eta}(t) + (y(\Delta t_0) - \bar{\eta}(\Delta t_0))
$$

$$
2\Delta t_0 \leq t < 3\Delta t_0,
\bar{y}(t) = \bar{\eta}(t) + (y(2\Delta t_0) - \bar{\eta}(2\Delta t_0))
$$

$$
:,
(k-1)\Delta t_0 \leq t < k\Delta t_0,
\bar{y}(t) = \bar{\eta}(t) + (y((k-1)\Delta t_0) - \bar{\eta}((k-1)\Delta t_0))
$$

(19)
As Fig. 14 shows, discrete prediction at this sampling rate is unable to model the noise. However, note that output correction was able to remove substantial bias.

4. Concluding remarks

This work presented continuous-time (CT) solutions for Hammerstein and Wiener systems with second-order plus-lead dynamic blocks when inputs are sinusoidal functions overlaid with step functions. Thus, to implement this algorithm in practice one needs to know the parameters of the input functions. These periodic parameters such as the frequency, the amplitude, and phase angle can be determined directly from modeling the input behavior, a task that may not be too difficult. However, the level of step changes still need to be distinguished from the periodic behavior. Thus, the practical implementation of this method requires an ability to determine step changes. The accuracy of this determination will affect the prediction accuracy of the proposed method (PM). Change point detection is an extensive area of research in statistics and process engineering and the text book by Basseville and Nikiforov (1993) gives an excellent overview.

A critical application of this work is the ability to model input process noise and translate it to accurate modeling of output noise. Since sinusoidal input noise is continuous, the modeling of this type of input noise, strictly from input data, requires a CT solution for structures that can represent nonlinear dynamic behavior. As demonstrated in this work, the proposed method is capable of meeting this requirement with high accuracy. In this work we presented algorithms to correct prediction bias using outputs sampled at equally spaced intervals. When outputs are correlated in time and are not measured at equally spaced intervals, the correlation algorithm that we presented will not be applicable. The treatment of correlated noise behavior of outputs under unequally spaced sampling times requires stochastic CT modeling of noise behavior which has received little attention in system identification. In the near future we hope to present a method that initiates research in this area.

References


