

Stat 416 Homework 3 Solutions

1. (a) (2 points) Blocking is not used in this experiment. Blocking was defined in our notes as “grouping similar experimental units together and assigning different treatments within such groups of experimental units.” Thus the pens of pigs are not blocks because all pigs in a pen received the same treatment.
- (b) (2 points) Each pen of pigs is an experimental unit because the treatments were randomly assigned and independently applied to the pens.
- (c) (2 points) An observational unit would be a muscle sample from a hog. There is a one-to-one correspondence between hogs and observational units.
- (d) (3 points)

$$Y_{ijk} = \mu + \delta_i + p_{ij} + e_{ijk}$$

where Y_{ijk} is the response for the k^{th} hog in the j^{th} pen that received treatment i ; μ is the overall mean; δ_i is the effect of the i^{th} diet; p_{ij} is the random effect of the j^{th} pen for the i^{th} diet; and e_{ijk} is the residual random effect for the k^{th} hog in the j^{th} pen that received treatment i . The p_{ij} are assumed to be independent and normally distributed with mean 0 and variance σ_p^2 . The e_{ijk} are independent and normally distributed with mean 0 and variance σ_e^2 . All the random effects are independent of one another. In abbreviated form, we have

$$Y = \text{diet pen.}$$

- (e) (2 points) This is a completely randomized design (with multiple observations per experimental unit).
2. (a) (2 points) diet and dose of drug
 - (b) (2 points) diet: A, B
dose of drug: 0, 10, 20, 30 mg/kg body weight
 - (c) (4 points) There are two types of experimental units in this example. Pens are the experimental units with respect to the treatment factor diet because diets were randomly assigned and independently applied to pens. Hogs are the experimental units with respect to the treatment factor dose of drug because the drug dose was randomly assigned and independently applied to individual hogs.
 - (d) (2 points) Based on the answer to (c), this experiment is best described as a split-plot design. The whole-plot factor is diet. The whole-plot experimental units are pens. The split-plot factor is dose of drug. The split-plot experimental units are hogs.
 - (e) (2 points) Y=diet dose diet:dose pen

(f) (4 points) Four pairs of pens should be formed, where each pair has one pen that received diet A and one that received diet B. Within each pair of pens, an arrow should connect the hogs from different pens that received the same dose of the drug (0 hog to 0 hog, 10 hog to 10 hog, etc.). There are several different ways for choosing the arrow direction that are equally valid. For each dose, half the arrows should point from a diet A pig to a diet B pig. Half should point from a diet B pig to a diet A pig.

(g) (4 points) Y=diet dose diet:dose dye pen slide

3. (4 points) Below is one correct answer. Other answers are possible. Each of the three comparison types (A vs. B, A vs. C, and B vs. C) need to appear on three slides. Each treatment needs to have 3 experimental units dyed with each dye.

A --> B B --> C C --> A

A <-- B B <-- C C <-- A

A --> B B --> C C --> A

(a) i. (1 point) $3\mu_1 - 9$

ii. (1 point) $\mu_1 + \mu_2 + \mu_3$

iii. (1 point) $2\mu_1 - \mu_2 + .5\mu_3$

(b) (2 points) $E(c) = c$ and $\text{Var}(c) = 0$.

(c) i. (2 points) $2\sigma^2$

ii. (2 points) $2\sigma^2$

iii. (2 points) $\sigma^2/2$

iv. (2 points) $4\sigma^2$

v. (2 points) σ^2

vi. (2 points) $4\sigma^2$

(d) i. (2 points) $n\sigma^2$

ii. (2 points) σ^2/n

(e) i. (2 points) 0

ii. (2 points) $\text{Cov}(Y, Y) = \text{Var}(Y)$

iii. (2 points) $\text{Cov}(Y_1, Y_2) = \text{Cov}(Y_2, Y_1)$

(f) i. (2 points) $\sigma_1^2 - \sigma_{12}$

ii. (2 points) $\sigma_1^2 - \sigma_2^2$

iii. (2 points) $3\sigma_{13} - 6\sigma_{12} + 4\sigma_{23} - 8\sigma_2^2$

(g) i. (2 points) $\text{Var}(Y_{111}) = \text{Var}(\mu + \tau_1 + m_{11} + e_{111}) = \text{Var}(m_{11} + e_{111}) = \sigma_m^2 + \sigma_e^2$

ii. (3 points)

$$\begin{aligned}\text{Cov}(Y_{ij1}, Y_{ij2}) &= \text{Cov}(\mu + \tau_i + m_{ij} + e_{ij1}, \mu + \tau_i + m_{ij} + e_{ij2}) \\ &= \text{Cov}(m_{ij} + e_{ij1}, m_{ij} + e_{ij2}) \\ &= \text{Cov}(m_{ij}, m_{ij}) + \text{Cov}(m_{ij}, e_{ij2}) + \text{Cov}(e_{ij1}, m_{ij}) + \text{Cov}(e_{ij1}, e_{ij2}) \\ &= \text{Cov}(m_{ij}, m_{ij}) \\ &= \text{Var}(m_{ij}) = \sigma_m^2\end{aligned}$$

iii. (2 points)

$$\frac{\sigma_m^2}{\sigma_m^2 + \sigma_e^2}$$

iv. (2 points) 0

v. (2 points) 0

vi. (2 points) $\text{Var}(\bar{Y}_{ij.}) = \text{Var}(m_{ij} + \bar{e}_{ij.}) = \sigma_m^2 + \sigma_e^2/2$

vii. (2 points) $\text{Var}(\bar{Y}_{i..}) = \text{Var}(\bar{m}_{i.} + \bar{e}_{i..}) = \sigma_m^2/4 + \sigma_e^2/8$

viii. (2 points) $\sigma_m^2/2 + \sigma_e^2/4$

(h) There are many ways to accomplish this part in R. These solutions show a couple of ways.

i. (3 points)

```
d=read.delim("http://www.public.iastate.edu/~dnett/microarray/hw3data.txt")
d=as.matrix(d)
m=matrix(d[,3],byrow=T,ncol=2)
mouse.vars=apply(m,1,var)
mouse.vars
[1] 0.00845 0.00245 0.00080 0.01445 0.00125 0.50000 0.04805 0.08820
```

OR

```
mouse.vars=tapply(d$y, factor(d$mouse), var)
mouse.vars
      1      2      3      4      5      6      7      8
0.00845 0.00245 0.00080 0.01445 0.00125 0.50000 0.04805 0.08820
mean(mouse.vars)
0.08295625
```

ii. (6 points)

```
mouse.means=apply(m,1,mean)
mouse.means
[1] 7.575 7.335 7.420 6.115 4.185 6.710 5.115 4.470
```

OR

```
mouse.means=tapply(d$y, factor(d$mouse), mean)
```

```
mouse.means
```

```
      1      2      3      4      5      6      7      8  
7.575 7.335 7.420 6.115 4.185 6.710 5.115 4.470
```

```
t.test(mouse.means[1:4], mouse.means[5:8], var.equal=T)
```

```
t = 3.0314, df = 6, p-value = 0.02306
```

iii. (4 points)

```
(var(mouse.means[1:4]) + var(mouse.means[5:8])) / 2
```

```
0.8629698
```

An estimate of $\sigma_m^2 + \sigma_e^2/2$ is 0.8629698. Thus an estimate of σ_m^2 is

$$0.8629698 - 0.08295625/2 = 0.8214917.$$