Some Matrix Basics

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A matrix is a rectangular array of numbers.

\[
\begin{bmatrix}
43 & 2 \\
18 & 7 \\
6 & 42
\end{bmatrix}
\]

We say the dimensions of this matrix are 3 x 2 because it has 3 rows and 2 columns.

A matrix with the same number of rows as columns is called a square matrix.

A matrix with only one column is called a vector.

\[
\begin{bmatrix}
8 \\
3 \\
-1 \\
7
\end{bmatrix}
\]

The transpose of a matrix is obtained by exchanging the rows and columns. The transpose of a matrix \( M \) is denoted \( M' \).

\[
\begin{bmatrix}
1 & 2 \\
3 & 4 \\
5 & 6
\end{bmatrix}' =
\begin{bmatrix}
1 & 3 & 5 \\
2 & 4 & 6
\end{bmatrix}
\]

Matrix Addition

\[
\begin{bmatrix}
2 & 9 \\
1 & 2
\end{bmatrix} +
\begin{bmatrix}
4 & 6 \\
-8 & 1
\end{bmatrix} =
\begin{bmatrix}
6 & 15 \\
-7 & 3
\end{bmatrix}
\]

Defined for matrices with identical dimensions.

Matrix Multiplication

\[
\begin{bmatrix}
1 & 2 \\
3 & 4 \\
5 & 6
\end{bmatrix} \\
\begin{bmatrix}
7 & 8 \\
9 & 10
\end{bmatrix} =
\begin{bmatrix}
1\times7+2\times9 & 1\times8+2\times10 \\
3\times7+4\times9 & 3\times8+4\times10 \\
5\times7+6\times9 & 5\times8+6\times10
\end{bmatrix}
\begin{bmatrix}
25 & 28 \\
57 & 64 \\
89 & 100
\end{bmatrix}
\]

Number of columns of the first matrix must match the number of rows of the second matrix.

Multiplication of a matrix by a single number:

\[
5 \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix} =
\begin{bmatrix}
5\times1 & 5\times2 \\
5\times3 & 5\times4
\end{bmatrix} =
\begin{bmatrix}
5 & 10 \\
15 & 20
\end{bmatrix}
\]
The Identity Matrix

A square matrix with ones on the main diagonal and zeros elsewhere is called the identity matrix. Such a matrix is usually denoted $I$. The $3 \times 3$ identity matrix is

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$AI = A$ and $IB = B$ for any matrices $A$ and $B$ of appropriate dimensions.

The Inverse of a Matrix

If $A$ is a square matrix and there exists a matrix $B$ such that $AB = I$, then $B$ is called the inverse of the matrix $A$. Usually we denote the inverse of a matrix $A$ by $A^{-1}$.

$$\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.50 & 0.00 \\ -0.25 & 0.50 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus, $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.50 & 0.00 \\ -0.25 & 0.50 \end{bmatrix}$

Some Matrix Operations in R

- `t(A)` gives the transpose of a matrix $A$.
- `A+B` computes the sum of the matrices $A$ and $B$.
- `A%*%B` computes $AB$, the product of the matrices $A$ and $B$.
- `solve(A)` gives the inverse of a square matrix $A$ if the inverse exists.
- `det(A)` computes the determinant of a square matrix $A$. 

```