STAT 611 **Fall 2012** **HW 5**

Due: Tuesday, October 23

1. Prove Result A.17.
2. Complete exercise A.72 from the text.
3. Suppose $\underset{n\times n}{\boldsymbol{A}}$ is a symmetric positive definite matrix. Prove that $\boldsymbol{A}$ is nonsingular.
4. Suppose $\underset{n\times n}{\boldsymbol{V}}$ is a symmetric positive definite matrix. Prove that

$$\boldsymbol{X}(\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X})^{-}\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X} = \boldsymbol{X}.$$

5. Suppose $\boldsymbol{X} = [\boldsymbol{X}_1, \boldsymbol{X}_2]$ is full-column rank. Suppose $\boldsymbol{X}_1'\boldsymbol{X}_2 \neq \boldsymbol{0}$.
   (a) Prove that $\boldsymbol{X}_2'(\boldsymbol{I} - \boldsymbol{P}_{\boldsymbol{X}_1})\boldsymbol{X}_2$ is a symmetric positive definite matrix.
   (b) Prove that

$$(\boldsymbol{X}_1'\boldsymbol{X}_1)^{-1}\boldsymbol{X}_1'\boldsymbol{X}_2(\boldsymbol{X}_2'(\boldsymbol{I} - \boldsymbol{P}_{\boldsymbol{X}_1})\boldsymbol{X}_2)^{-1}\boldsymbol{X}_2'\boldsymbol{X}_1(\boldsymbol{X}_1'\boldsymbol{X}_1)^{-1}$$

   is nonnegative definite.
6. Suppose $x_1, \ldots, x_n > 0$. Prove that

$$\frac{\sum_{i=1}^n x_i^3}{(\sum_{i=1}^n x_i^2)^2} \geq \frac{1}{\sum_{i=1}^n x_i}$$

   with equality iff $x_1 = \cdots = x_n$. (We know this must be true from the last slides of our notes on the Aitken Model, but prove the result directly.)
7. Complete exercises 4.20–4.23 in the text.