

1. Consider the model $\underline{y} = X\underline{\beta} + \underline{\varepsilon}$ where $\underline{\varepsilon} \sim N_T(\underline{0}, \sigma^2 I)$. Suppose $\text{rank}(L_{K \times q}) = q$ and $L'\underline{\beta}$ is estimable. Let M be any other full-column rank matrix such that $L'\underline{\beta} = \underline{0} \iff M'\underline{\beta} = \underline{0}$.

Prove that

$$\frac{(L'\hat{\underline{\beta}})'(L'(X'X)^{-1}L)^{-1}L'\hat{\underline{\beta}}}{q\hat{\sigma}^2} = \frac{(M'\hat{\underline{\beta}})'(M'(X'X)^{-1}M)^{-1}M'\hat{\underline{\beta}}}{q\hat{\sigma}^2}.$$

2. Recall that the Working-Hotelling $1 - \alpha$ confidence band for the mean function in simple linear regression is given by

$$\hat{\beta}_0 + \hat{\beta}_1 x \pm \sqrt{2(F_{2,T-2,1-\alpha})\hat{\sigma}^2 \left[\frac{1}{T} + \frac{(x - \bar{x})^2}{\sum_{t=1}^T (x_t - \bar{x})^2} \right]}.$$

Prove that

$$P(\beta_0 + \beta_1 x \text{ contained in the confidence band } \forall x \in \mathbb{R}) = 1 - \alpha.$$

3. Consider the model $\underline{y} = X\underline{\beta} + \underline{\varepsilon}$ where $\underline{\varepsilon} \sim N_T(\underline{0}, \sigma^2 I)$. Suppose $\text{rank}(L_{K \times q}) = q$ and $L'\underline{\beta} = \underline{\tau}$ is estimable. Show that the level α F -test of $H_0 : \tau_1 = \dots = \tau_q$ will lead to rejection of H_0 if and only if there exists a contrast $c_1\tau_1 + \dots + c_q\tau_q$ whose simultaneous $1 - \alpha$ Scheffé interval excludes 0.
4. Consider the model $\underline{y} = X\underline{\beta} + \underline{\varepsilon}$ where $\underline{\varepsilon} \sim N_T(\underline{0}, \sigma^2 I)$. Provide an expression for a $100(1 - \alpha)\%$ confidence interval for σ^2 .