STAT 611 **Fall 2012** **HW 4**

Due: Thursday, October 11

1. Prove the following: If $\mathcal{S}$ and $\mathcal{T}$ are vector spaces in $\mathbb{R}^n$ with $dim(\mathcal{S}) < dim(\mathcal{T})$, then there always exists a nonzero vector $\boldsymbol{x} \in \mathcal{T}$ such that $\boldsymbol{x} \in \mathcal{S}^{\perp}$.

2. Suppose $\boldsymbol{A}$ is an $m \times n$ matrix.

   (a) Prove that there exists a matrix $\boldsymbol{G}$ such that $\boldsymbol{A}'\boldsymbol{A}\boldsymbol{G} = \boldsymbol{A}'$.

   (b) Suppose $\boldsymbol{G}$ satisfies $\boldsymbol{A}'\boldsymbol{A}\boldsymbol{G} = \boldsymbol{A}'$. Complete the following statement and prove that your answer is correct.

$$\boldsymbol{A}\boldsymbol{G} = \boldsymbol{I} \iff \underline{\hspace{3cm}}.$$

3. Complete exercises 3.22, 3.26, 4.1, 4.2, 4.11, 4.12 from the text.