

STAT 611

FALL 2011

HW 2

DUE TUESDAY, SEPTEMBER 13

1. Suppose  $A$   $m \times n$ . PROVE THAT THE  
MAXIMUM NUMBER OF LI ROWS OF  
 $A$  (DENOTED  $r$ ) IS THE SAME AS  
THE MAXIMUM NUMBER OF LI COLUMNS  
OF  $A$  (DENOTED  $c$ ). WE SKIPPED  
A PROOF OF THIS IN CLASS, BUT IT  
IS NOT TOO INVOLVED. ONE PROOF  
GOES LIKE THIS.  $\exists$  MATRICES  $L$   $m \times r$  AND

$R$   $r \times n$   $\Rightarrow LR = A$ . (WHY?) LIKEWISE,  $\exists$

MATRICES  $C$   $m \times c$  AND  $M$   $c \times n$   $\Rightarrow CM = A$ .

(WHY?)

Now, WHAT DOES  $L R = A$   
 $m \times r$   $r \times n$

TELL YOU ABOUT  $c$  RELATIVE TO  $r$ ?

WHAT DOES  $C M = A$  TELL YOU  
 $m \times c$   $c \times n$

ABOUT  $r$  RELATIVE TO  $c$ ?

2. PROVE THAT  $\forall$  MATRIX  $A$  OF  
 $m \times n$

RANK  $r$ ,  $\exists$  NONSINGULAR MATRICES

$$P \text{ AND } Q \Rightarrow P A Q = \begin{bmatrix} I_{r \times r} & 0 \\ 0 & 0 \end{bmatrix}.$$

3. SUPPOSE  $\text{RANK}(A) = n$  AND

$m \times n$

$\text{RANK}(C) = k$ . THEN

$k \times n$

$\text{RANK}(B) = \text{RANK}(ABC)$ .

$n \times k$

PROVE THIS RESULT.

4. PROVE PARTS (a) AND (b) OF RESULT A.17 AS STATED IN COURSE NOTES ON TRACE, DETERMINANTS, AND MATRIX FACTORIZATION.

5. FIND THE DETERMINANT OF

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

USING BOTH DEFINITIONS GIVEN IN COURSE NOTES.

6. PROVE THAT IF  $A$  HAS FULL  
 $m \times n$  COLUMN RANK, THEN  $A^T A$  IS  
NONSINGULAR.

7. COMPLETE THE FOLLOWING  
PROBLEMS FROM THE APPENDIX.

A. 22, 24, 25, AND 32.