STAT611 **Exam 1** **September 27, 2011**

1. Is it true that every symmetric matrix has at least one symmetric generalized inverse? Provide a proof to support your answer.

2. Suppose $\mathcal{S}$ is a vector space in $I\!R^n$. Complete the following statement and prove that it is true.

$$\mathcal{S} \bigcup \mathcal{S}^{\perp} = I\!R^n \iff \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

3. Prove that $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$ for all $m \times n$ matrices A and B.

4. Suppose $A\underline{x} = \underline{c}$ is a consistent system of equations. We have a result that characterizes the set of all solutions to $A\underline{x} = \underline{c}$. Without using that result, prove that $A\underline{x} = \underline{c}$ has a unique solution iff A has full-column rank.

5. Suppose you are given an $n \times p$ matrix X and an $n \times 1$ vector $\underline{y}$.

   (a) State the normal equations.

   (b) Provide the set of all possible solutions to the normal equations.

   (c) Respond to the following question, using formal proof to support your answer. Is the set of all possible solutions to the normal equations a vector space?

6. Suppose $E(y_{ij}) = \mu + \alpha_i + \tau_j$ for $i = 1, 2; j = 1, 2$. Let

$$\underline{y} = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{22} \end{bmatrix} \quad \text{and} \quad \underline{\beta} = \begin{bmatrix} \mu \\ \alpha_1 \\ \alpha_2 \\ \tau_1 \\ \tau_2 \end{bmatrix}.$$

   (a) Find the least-squares estimator of $E(\underline{y})$.

   (b) Now suppose y_{22} is missing so that

$$\underline{y} = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{21} \end{bmatrix}.$$

   Find the least-squares estimator of $E(\underline{y})$.